Thermal entanglement in a three particles system

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Abstract: *Entanglement of quantum systems changes under environmental conditions like temperature, magnetic field, noise and so on. In this paper the thermal entanglement in a spin chain with three particles by exchange interaction is investigated in terms of the measure of entanglement called 'negativity'.*
We find that the entanglement is decreased with increasing temperature; entanglement is increased with increasing change interaction. Also we find that the entanglement maybe is enhanced under a uniform magnetic field with increasing magnetic field in constant temperature.

Keywords: Exchange Interaction, Negativity, Magnetic field, Thermal entanglement.

1. Introduction:

Entanglement is the behavior of some quantum systems which first interacts each other and then separate. In quantum mechanics knowing about one part leads to have information about the other one. At first entanglement just was a theoretical subject but now it has many application in quantum information and computation such as teleportation [1], super dense coding[2], quantum computation [3,4] and some cryptographic protocols [5,6] and so on.

There are several definitions of entanglement measure of system .In this paper, we use negativity $N(\rho)$ which was shown to be an easily computable measure pure as well as mixed states [7]. In definition is based on the trace norm of the particle of ρ^{T_A} of bipartite mixed state ρ. Negativity as an entanglement measure is motivated by the Peres-Horodecki positive partial transpose separability criterion and is computed as follows:

$$N(\rho)=\frac{\left\|\rho^{T_A}\right\|_1-1}{2} \qquad (1)$$

with this definition $N(\rho)$ is the absolute value of the sum of the negativity eigenvalues of ρ^{T_A} .Where ρ^{T_A} is partial transpose of ρ with respect to A party.

2. The model:

The state of the system at thermal equilibrium at temperature T is characterized by the thermal density matrix $\rho(T)=\frac{1}{Z}\exp\left(\frac{-H}{K_BT}\right)$ where $Z=\mathrm{Tr}\left(\exp\left(\frac{-H}{K_BT}\right)\right)$ is the partition function and K_B is Boltzmann constant. The entanglement in $\rho(T)$ is called thermal entanglement [8].

In This paper we consider a chain composed of three spins which two particles are spin-half (particle numbered one and three) and second particle has spin-one. For

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this system under external magnetic fields and with nearest neighbor interaction and next nearest neighbor interaction, the Hamiltonian is [9]

$$H = J_1(S_1.S_2) + J_2(S_2.S_3) + \sum_{i=1}^{3} B_i S_{iz} \tag{2}$$

In which the neglected exchange coupling term along the Z-axes is assumed to be much smaller than the coupling the X-Y plane, the magnetic field is assumed to be along the Z-axes, J_1 and J_2 are the coupling constants nearest neighbor and next nearest neighbor. We investigate the thermal entanglement of this system with a uniform and nonuniform magnetic field.

3. Uniform magnetic field

$$\vec{B}_1 = \vec{B}_2 = \vec{B}_3 = B\hat{Z}$$

To evaluate the thermal entanglement, we first of find the eigenvalues and corresponding eigenstates of the Hamiltonian which are seen to be:

$$E_1 = \frac{1}{4}\mathrm{J}_2 - \mathrm{B} + \frac{1}{4}\sqrt{\alpha}$$

$$E_2 = \frac{1}{4}\mathrm{J}_2 - \mathrm{B} - \frac{1}{4}\sqrt{\alpha}$$

$$E_3 = \frac{1}{4}\mathrm{J}_2 + \mathrm{B} + \frac{1}{4}\sqrt{\alpha}$$

$$E_4 = \frac{1}{4}\mathrm{J}_2 + \mathrm{B} - \frac{1}{4}\sqrt{\alpha}$$

$$E_5 = -\frac{1}{2}\mathrm{J}_2 + \mathrm{B}$$

$$E_6 = -\frac{1}{2}\mathrm{J}_2 - \mathrm{B}$$

$$E_7 = 2B$$

$$E_8 = -2B$$

$$E_9 = \frac{1}{4}\mathrm{J}_2 + \frac{1}{4}\sqrt{\beta}$$

$$E_{10} = \frac{1}{4}\mathrm{J}_2 - \frac{1}{4}\sqrt{\beta}$$

$$E_{11} = -\frac{1}{2}\mathrm{J}_2$$

$$E_{12} = \circ$$

Where

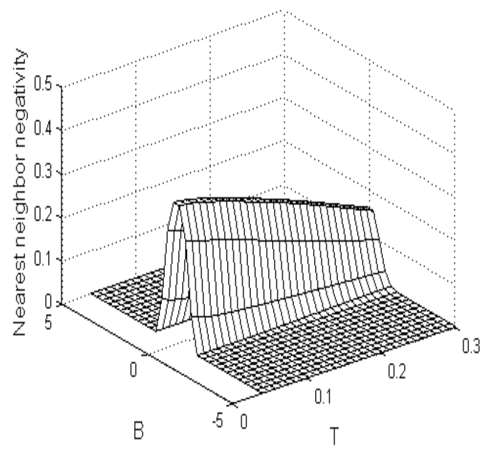
$$\alpha = J_2^2 + 16J_1^2$$

$$\beta = J_2^2 + 32J_1^2$$

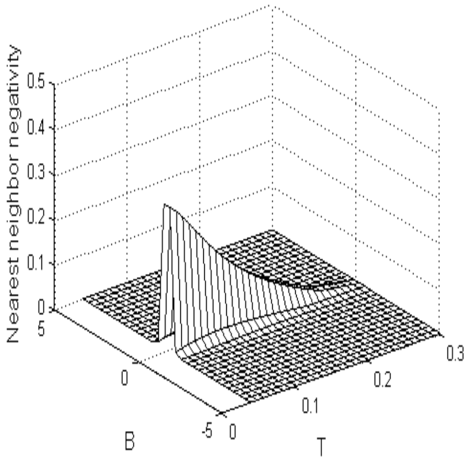
Then with use of these eigenvalues and eigenvectors, we construct the total density matrix (ρ_{123}), and evaluate the entanglement between nearest neighbor and next nearest neighbor.

3.1 Nearest neighbor entanglement

In this case, we evaluate negativity with use of the density matrix $\rho_{12} = Tr_3(\rho_{123})$ and then we plot nearest neighbor negativity in terms of temperature and magnetic field for J_1=1 and a) J2=0.5 b) J2=1.5



(1.a)



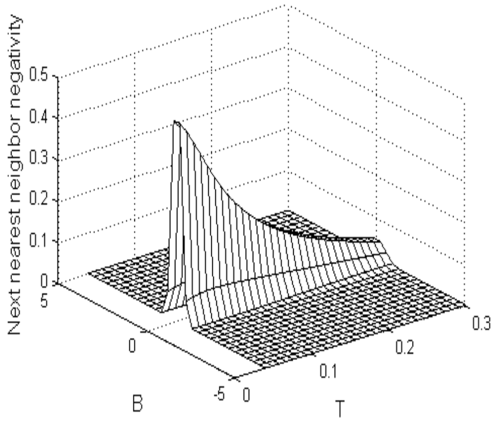
(1.b)

Fig1.The nearest neighbor negativity in terms of B and T for J_1=1 and a)J_2=0.5 b)J_2=1.5

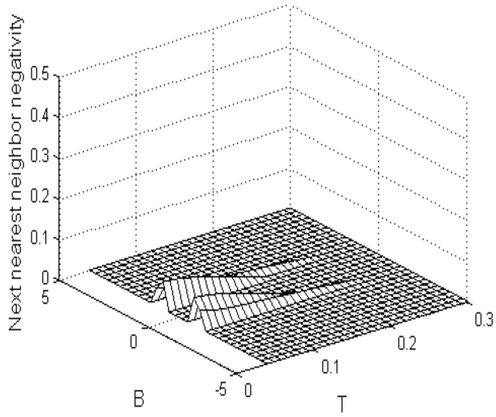
Figure 1 shows that with increasing temperature, entanglement is decreases also entanglement is decreased with increasing magnetic field in constant temperature. Furthermore with increasing J_2 the amount of entanglement decreases and the region which has nonzero entanglement become smaller. For any temperature the entanglement is symmetric with respect to zero magnetic fields.

3.2 Next nearest neighbor entanglement

We evaluate negativity with use of the density matrix $\rho_{13} = Tr_2(\rho_{123})$. We show our calculation result in fig 2 for J_2=1 and a) J1=0.5 b) J1=1.5



(2.a)



(2.b)

Fig2.The nearest neighbor negativity in terms B and T for J_2=1 and a) J_1=0.5 b) J_1=1.5.

We can see that with increasing temperature, the value and the region of entanglement becomes smaller, and with increasing of magnetic field, entanglement is decreased in constant temperature. With increasing J_1 (in equal T and B), entanglement is decreased. In fig 2(a), there is one peak and for any temperature entanglement is symmetric with respect to zero magnetic fields, also with increasing temperature, entanglement is decreased. But in fig 2(b) we can see two peaks, with increasing temperature, the left and right peak, disappear.

4. Nonuniform magnetic field $\vec{B}_1 = -\vec{B}_2 = \vec{B}_3 = B\hat{Z}$

In this case, eigenvalues of Hamiltonian are given by

$$E_1 = E_2 = \frac{1}{4}\mathrm{J}_2 + \frac{1}{4}\sqrt{a_1}$$

$$E_3 = E_4 = \frac{1}{4}\mathrm{J}_2 - \frac{1}{4}\sqrt{a_2}$$

$$E_5 = \frac{1}{6}\sqrt[3]{a_4} - 6\left(-4/3B^2 - 2/3J_1^2 - 1/36\,J_2^2\right) / \left(1/6J_2 + \sqrt[3]{a_4}\right)$$

$$E_6 = \frac{1}{6}\sqrt[3]{\mathrm{a}_4} - 6\frac{\left(-4/3B^2 - 2/3J_1^2 - 1/36J_2^2\right)}{\left(1/6J_2 + \sqrt[3]{a_4} + 1/2\sqrt{3}i\left(1/6\sqrt[3]{a_4} + 6\left(-4/3B^2 - 2/3J_1^2 - 1/36J_2^2\right)\right)/\sqrt[3]{a_4}\right)}$$

$$E_7 = \frac{1}{6}\sqrt[3]{\mathrm{a}_4} - 6\frac{\left(-4/3B^2 - 2/3J_1^2 - 1/36J_2^2\right)}{\left(1/6J_2 + \sqrt[3]{a_4} - 1/2\sqrt{3}i\left(1/6\sqrt[3]{a_4} + 6\left(-4/3B^2 - 2/3J_1^2 - 1/36J_2^2\right)\right)/\sqrt[3]{a_4}\right)}$$

$$E_8 = -\frac{1}{2}\mathrm{J}_2 - \mathrm{B}$$

$$E_9 = -\frac{1}{2}J_2 + B$$

$$E_{10} = -\frac{1}{2}J_2$$

$$E_{11} = E_{12} = \circ$$

Where

$$a_1 = J_2^2 + 16J_1^2 - 8BJ_2 + 16B^2$$

$$a_2 = J_2^2 + 16J_1^2 + 8BJ_2 + 16B^2$$

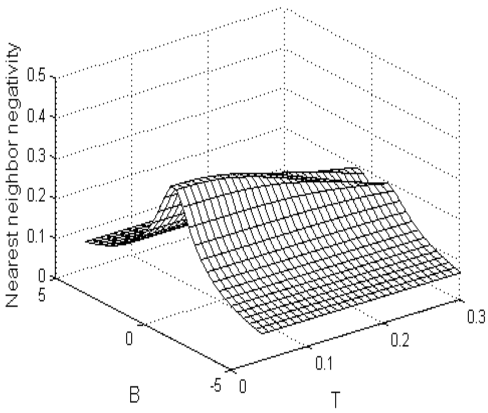
$$a_3 = -768B^6 - 1152J_1^2B^4 + 96B^4J_2^2 - 576J_1^4B^2 - 120J_1^2J_2^2B^2 - 3J_2^4B^2 - 96J_1^6 - 3J_2^2J_1^4$$

$$a_4 = -144J_2B^2 + 36J_2J_1^2 + J_2^3 + 12\sqrt{a_3}$$

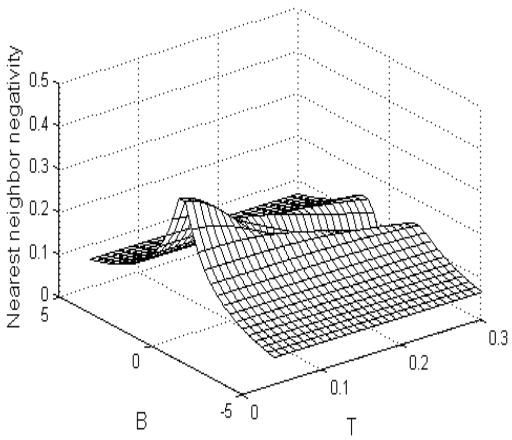
As the same as former case, we investigate entanglement for nearest neighbor and next nearest neighbor.

4.1 Nearest neighbor entanglement

We plot fig 3 with the same condition in fig 1



(3.a)



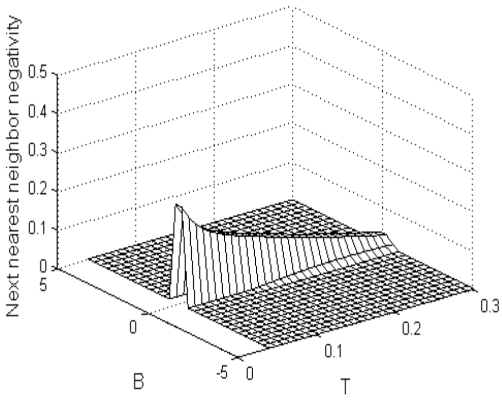
(3.b)

Fig3.the next nearest neighbor negativity in terms B and T for J_1=1 and a) J_2=0.5 b) J_2=1.5.

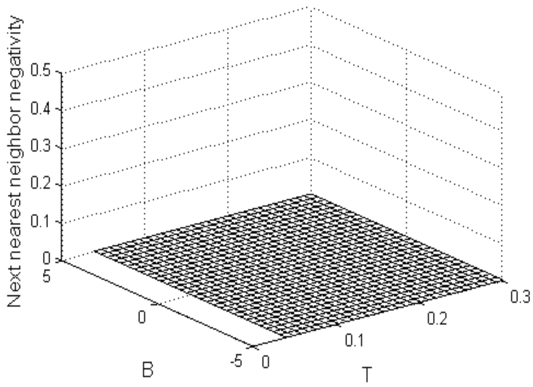
Figure 3 shows that with increasing temperature, the value and region of entanglement (in same B and J) become smaller, with increasing magnetic field , entanglement is increased at the high temperature, and with increasing J_2 (in same T and B) entanglement is decreased.

4.2 Next nearest neighbor entanglement

We plot fig 4 with the same condition in fig 2



(4.a)



(4.b)

Fig4. the next nearest neighbor negativity in terms B and T for J_2=1 and a) J1=0.5 b) J1=1.5.

We can see that with increasing temperature, magnetic field and J_1, entanglement is decreased.

5. Conclusion

In this paper, we focus our attention on a spin chain with two spin one-half and one spin one, and investigate the affect of temperature, magnetic field and exchange interaction on the entanglement in uniform and nonuniform magnetic field. We find that in both cases ($\vec{B}_1 = \vec{B}_2 = \vec{B}_3 = B\hat{Z}$ and $\vec{B}_1 = -\vec{B}_2 = \vec{B}_3 = B\hat{Z}$), with increasing temperature, entanglement is decreased .With increasing magnetic field, entanglement is decreased (with the exception of case nearest neighbor $\vec{B}_1 = -\vec{B}_2 = \vec{B}_3 = B\hat{Z}$) in constant temperature. Furthermore, entanglement between any of two particles, is increased with increasing their exchange interaction, but for the same particles is decreased with increasing the exchange interaction for two other particles and vice versa.

6. Acknowledgements

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7. References

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