Bernoulli's Law

Article Code:

Area: Fluid Mechanics

Keywords:

Background Knowledge:

When a fluid flows in a pipe, the pressure, *P,* and the velocity, *v*, of the fluid in two points, 1 and 2, of the pipe are connected as in Eq.1 below (**Bernoulli's law**):

$$P + \frac{\rho v_1^{\ 2}}{2} + g\rho z_1 = P_0 \ldots\ldots\ldots Eq.1$$

Equ.1 may take the form:

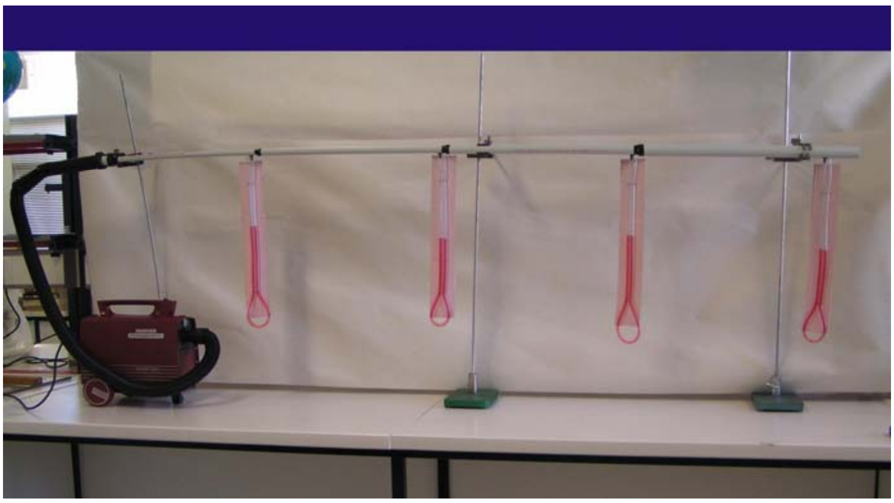
$$P - P_{atm} + \frac{\rho v_1^{\ 2}}{2} + g\rho z_1 = P_0 - P_{atm} \ldots\ldots\ldots Eq.2$$

In Eq.1, ρ is the density of the fluid, g is the gravitational constant (~9.81m/s^2) and z is the vertical distance from ground (height) of the corresponding point of the pipe. P_{atm} is the atmospheric pressure and P_0 a constant (it denotes the pressure of the fluid in a point with v=0 and z=0). This is known as Bernoulli's law and is valid if the flow is steady and non-turbulent and if the fluid is without viscosity and if the variations of the density of the fluid are negligible. These conditions are satisfied adequately enough for water. They are also adequately satisfied for air if the height difference do not induce significant density changes and if the velocity of the airflow is not too high (i.e. less than 200km/h; note that when the velocity of a wind exceeds 120km/h (or 65 knots) it is classified as a wind of force 12 in the Beaufort scale).

Eq.1 implies that along a horizontal pipe (z constant) through which a fluid is passing, the pressure, *P*, is less in places where the velocity, *v*, is higher, i.e. where the pipe's section is smaller (the density of the fluid was assumed constant).

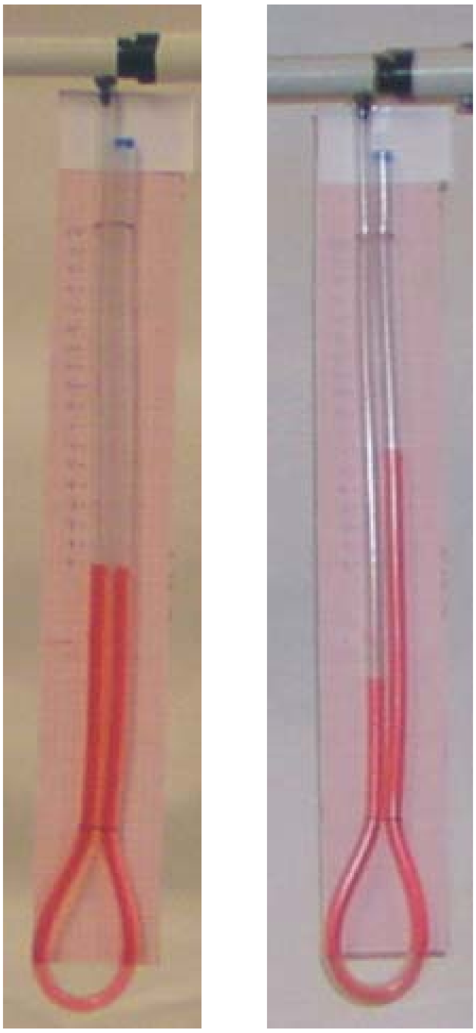
Experimental Construction:

Figure 1. Experimental construction



Plastic pipe segments (like the ones used in drainage) of different cross-sections were used to construct the equipment shown in Fig.1. Air tightness was obtained using plastic band (insulating band of electricians). A vacuum cleaner was used to produce an air flow through the tubes. Pressure and pressure differences is measured through the U-shaped transparent plastic tubes (it is used in levelling the cement floors during construction) and the coloured water within ('pressure gauges'-See **Figure 2**). Every U-shaped tube has one open end within a tube (pressure within the tube) the other open end outside (atmospheric pressure). When there is no air flow through the tubes both liquid legs of every U-shaped tube are level. With an air flow through the tubes a pressure difference appears which can be measured from the difference of the heights of the liquid in the U-shaped legs (the difference in the height of the coloured liquid, which may be positive or negative, measures the factor P-P_{atm}, see Equ. 2).

Figure 2. Pressure Gauges:
a-without air flow, b with air flow



Teaching Approaches:

1.-At first, with the vacuum cleaner inactive, students were asked to explain why the liquid levels in the 'pressure gauges' were the same, a task easily carried out.

2.-They were then asked what will happen if the vacuum cleaner is turned on sucking air to the left through the construction and explain their answers.

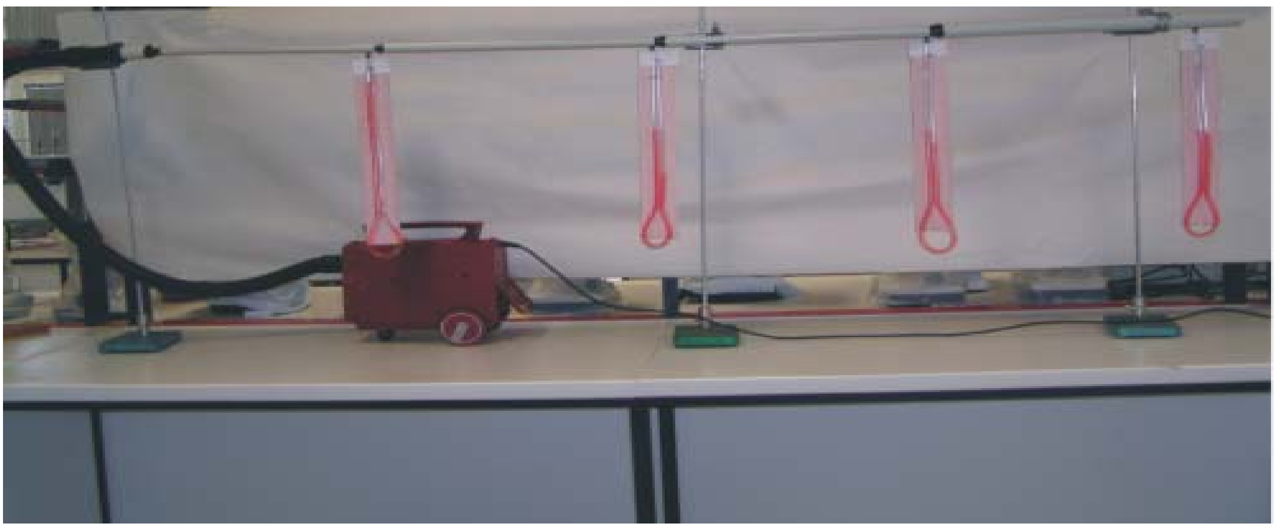
3.-The vacuum cleaner is turned on sucking air (air flow to the left). The coloured water in the legs with their open end inside the tubes is raised (see Figure 3). Students are asked to offer explanations.

4.-The discussion usually leads to the conclusion that what is observed is due to the sucking action of the vacuum cleaner which sucks also the liquid in the tubes.

5.-They are then asked what will happen if the tubes are connected to the other end of the vacuum cleaner so that air is blown to the tubes (with a direction from left to right). Almost always all of them agree that the blowing action of the air will lower the liquid in the legs with their open end inside the tubes.

6.-The experiment is carried out and they observe that the situation is actually similar to the one in step 3.

Figure 3. Air flow to the left



7.-The vacuum cleaner is then connected to the other end of the connected pipes.

8.-Students are asked what will happen if the vacuum cleaner is turned on with a sucking action (air flow towards the right). Referring to the air flow direction, the situation is similar to the one in step 6 but referring to blow-suck action the situation is similar to the one in step 3.

9.-Carrying out the experiment, a situation similar to the one in steps 3 and 6 is observed.

10.-Students are then asked what will happen if the vacuum cleaner is turned on with a blow action (air flow towards the left). Referring to the air flow direction the situation is similar to the one in step 3 but referring to blow-suck action the situation is similar to the one in step 6.

11.-Carrying out the experiment the following is observed (Figure 4). Comparing Figure 3 and Figure 4, a misunderstanding may arise if the signs of the elevetion of the liquid in the pressure gauges are not observed.

Figure 4. Air flow to the left / blow action

12.-Discussion on the observed (apparent) differences between the situations in steps 3, 6 and 9 on the one hand and the situation in step 11 may be very fruitful in understanding the phenomenon.

Tips:

Connection of pipes in increasing-decreasing order of cross-section.

In step 10 it could start the blow action gradually through a straggling action

End point of inside tubes about midway of the pipes and midway within each pipe

Instead of U-shaped tubes one tube in a bowl.

In step 11 it could also be demonstrated if first the levels are observed with the cleaner in operation but blocking the exit and then when the exit is not blocked.

Actually the situation is similar to the previous ones

Relevant issues: Equ.2 implies that the diagrams f(x,y) and g(h,y) (with $x=P-P_{atm}$, $y=1/d^4$, h=the height diference between the two branches of the U-shaped tubes and d the diameter of the pipe segments) should be represented by straight lines. From this diagram other values (e.g. P_o, v, ρ *or* *flow*) may be estimated. An example refring to case 3 (Figure 3) is given in the diagrama of Figure 5, where the elevation of the liquid in the pressure gauges is given as a function of $1/d^4$ where d is the diameter of the pipe segment).

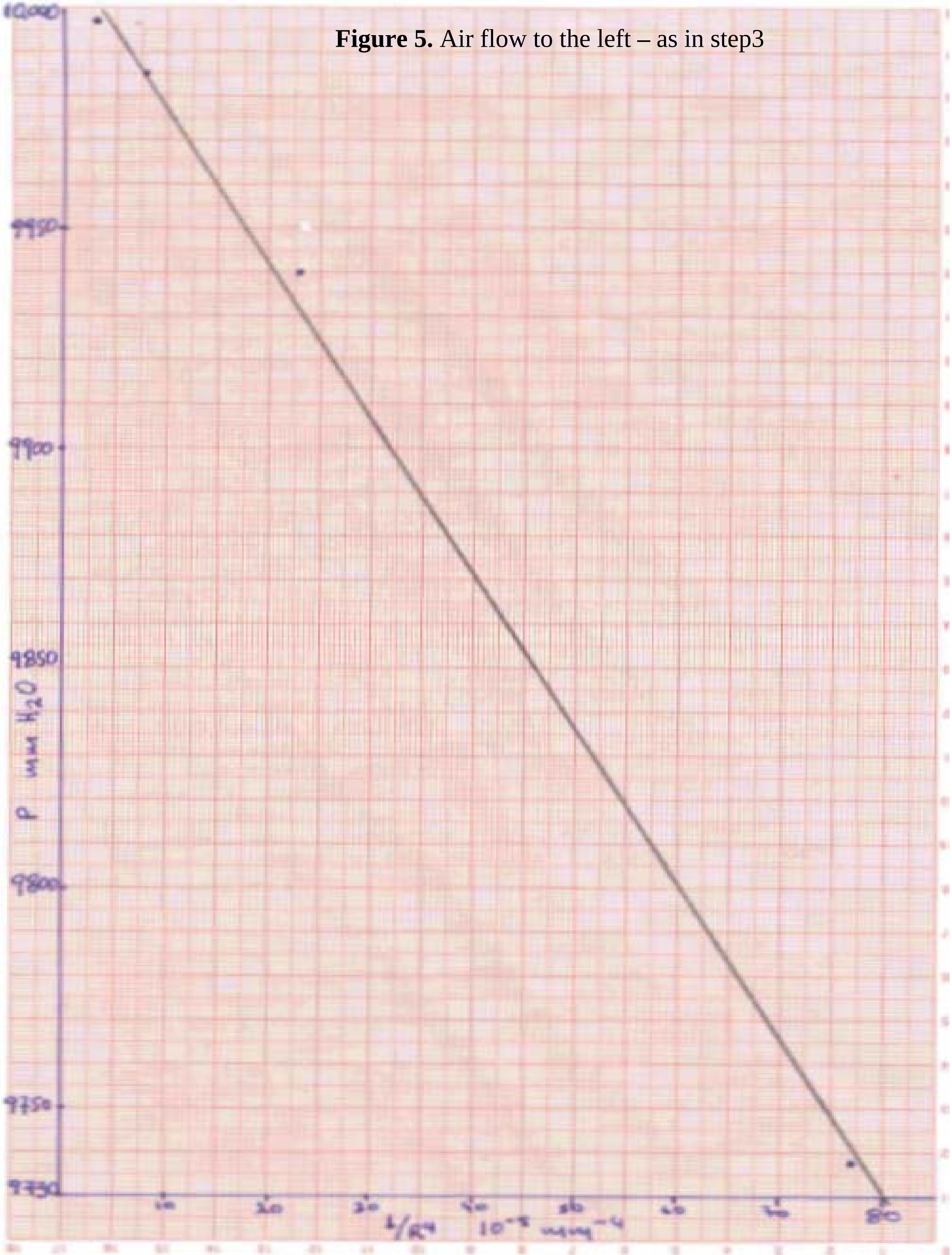


Figure 5. Air flow to the left – as in step3

Related Articles:

Further Reading:

The work in this article was partially financed by the European Commission (contract no. 110157-CP-1-2003-1-PT-COMENIUS-C3). The information and views expressed here reflect the authors. Neither the European Commission nor the authors may be liable for any use that may be made of this document.

Acknowledgements:

This construction was inspired from a demonstration within the University lectures of Professor D. Ploumbidis at the Department for Primary Education of the University of Crete.

Authors:

Prof. P. G. Michaelides, B.Sc., LL.B., PhD, e-mail: michail@edc.uoc.gr

Miltiadis Tsigris, chemist, mtsigris@edc.uoc.gr