Abstracts of activities during the ESU 5

The ESU 5 activities are **listed** alphabetically below and include
Plenary Lectures,
Panels,
3-hour Workshops based on historical and epistemological material,
2-hour workshops based on didactical pedagogical material
30-minute oral presentations and
10-min short oral presentations

All activities during the ESU 5 are related to one of its main themes:

MAIN THEMES

1. History and Epistemology as tools for an interdisciplinary approach in the teaching and learning of Mathematics and the Sciences
2. Introducing a historical dimension in the teaching and learning of Mathematics
3. History and Epistemology in Mathematics teachers' education
4. Cultures and Mathematics
5. History of Mathematics Education in Europe
6. Mathematics in Central Europe

The **abstracts** are shown below:
Plenary Lectures, Panels, 3-hour Workshops, 2-hour Workshops, Oral Presentations
Short Presentations

Plenary Lectures

(Ordered alphabetically)

Name	Title	Theme	Language	Country
Corry Leo	Axiomatics between Hilbert and R.L. Moore: Two Views on Mathematical Research and their Consequences on Education	1	English	Israel
Gispert Hélène, Schubring Gert	The History of Mathematics Education and its contexts in 20th century France and Germany	5	English	France, Germany
Hyksova Magdalena	Contribution of Czech Mathematicians to Probability Theory	6	English	Czech Republic
Puig Luis	Researching the history of algebraic ideas from an educational point of view	2	English	Spain
Rebstock Ulrich	Mathematics in the service of the Islamic community	4	English	Germany
Schweiger Fritz	The implicit grammar of mathematical symbolism	3	English	Austria

Plenary Lectures ABSTRACTS

(ordered by theme)

***Theme* 1**

AXIOMATICS BETWEEN HILBERT AND R.L. MOORE: TWO VIEWS ON MATHEMATICAL RESEARCH AND THEIR CONSEQUENCES ON EDUCATION

Leo Corry
University of Tel Aviv, Ramat Aviv 69978, Israel

David Hilbert is widely acknowledged as the father of the modern axiomatic approach in mathematics. The methodology and point of view put forward in his epoch making *Foundations of Geometry* (1899) had lasting influences on research and education throughout the twentieth century. Still, his own conception of the role of axiomatic thinking in mathematics and in science in general was significantly different from the way in which it

became to be understood and practiced by mathematicians of the following generations, including some who believed to be developing Hilbert's original line of thought. The topologist Robert L. Moore was prominent among those who put at the center of their research an approach derived from Hilbert's then recently introduced axiomatic methodology. Under his leadership a very unique and fruitful school of mathematics developed at the University of Texas beginning in the 1920s. Moore actively put forward his view according to which the axiomatic method would serve as a most useful teaching device in both graduate and undergraduate teaching in mathematics and as a tool for identifying and developing creative mathematical talent. The present talk is devoted to presenting two different approaches to the essence of the modern axiomatic method, that of Hilbert and that of Moore. It explores the historical process that led from the first to the second and the diverging educational implications that arise from each of them.
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***Theme* 2**

RESEARCHING THE HISTORY OF ALGEBRAIC IDEAS FROM AN EDUCATIONAL POINT OF VIEW

Luis Puig
Departament de Didàctica de la Matemàtica, Universitat de València, Spain

I will present some examples of investigations in the history of the algebraic ideas, which have been made with the purpose of being used in educational mathematics, and following two principles.
The first one is that the problématique of the teaching and learning of algebra is what determines what aspects and texts of the history of algebraic ideas are worth studying in depth, and which questions should be addressed to them.
The second one is a kind of Embedment Principle that asserts that signs, syntax, semantics and pragmatics of the algebraic language that students have to learn in school are bearers of the cognitive activity of previous generations (of mathematicians).
Investigating history from this point of view allows us to develop cognitive models, by looking at pupils' productions, behaviours or cognitions through the lenses that our study of historical texts provides us, and teaching models.
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***Theme* 3**

THE IMPLICIT GRAMMAR OF MATHEMATICAL SYMBOLISM

Fritz Schweiger
Interfakultärer Fachbereich Didaktik und LehrerInnenbildung, Universität Salzburg, Austria

The appearance of symbols is quite typical for mathematical texts. The historical development of the symbolic language is an interesting topic in itself. The correct use of mathematical signs follows 'grammatical' rules which very often are learned without being taught in an explicit way. The use of calculators, computer algebra, and word processors increases the awareness of such rules. Many of these rules are rooted in history and follow general semiotic principles. The following principles will be discussed: serialisation, similarity of configuration and alphabetic correspondence, separation, symmetry and duality, well formed strings, iconicity and semantic of symbols.
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***Theme* 4**

MATHEMATICS IN THE SERVICE OF THE ISLAMIC COMMUNITY

Ulrich Rebstock
University of Freiburg, Germany

Islamic mathematics that accumulated manifold influences from Greece, the iddle and the Far Esat, very early developed sub-scientific branches of teaching and application. From this practical tradition, the Muslin society was invited to profit, and, in return, the discipline itself to flourish. From among the literary tradition of this 'mathematics of practitioners' two-texts – one of a certain al-Uqlidisi, the second of the famous mathematician Abu-l-Wafa al-Buzgani (both 4th/10th century) will be shortly described. By offering their services to the community, these mathematicians inevidently touched upon sensitive social and religious issues. Some selected examples will demonstrate the productive effect between mathematical accuracy ans the legal needs of a Muslin society.
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***Theme* 5**

THE HISTORY OF MATHEMATICS EDUCATION AND ITS CONTEXTS IN 20TH CENTURY FRANCE AND GERMANY

Hélène Gispert
University of Orsay, France
Schubring Gert
University of Bielefeld, Germany

In this plenary session, some key moments in the development of the teaching of mathematics in two countries will be presented which decisively influenced the overall history in Europe: in France and Germany. Since the respective developments in France and in Germany mutually influenced one another, the presentation will be given jointly and in a dialogue mode.
Among these key moments in history, the period from 1902 to 1914 will highlight their interaction, since it not only comprises the beginning of international cooperation in mathematics education, but also decisive exchanges about goals and directions of reform and about the modernization of teaching mathematics. Another key moment will be the "modern math" movement. The mutual relation will in particular emphasize the imbedding of mathematics education, its contents and objectives into the cultural, economic and social contexts in these periods and countries.
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***Theme* 6**

CONTRIBUTION OF CZECH MATHEMATICIANS TO PROBABILITY THEORY

Magdalena Hyksova
Faculty of Transportation Sciences, Department of Applied Mathematics,
Czech Technical University in Prague, Czech Republic

The lecture discusses the most important contributions of Czech mathematicians to the theory probability and its interpretations. It recalls the theory proposed by Bernard Bolzano and its possible influence on further development of logical interpretation of probability. Then the contribution of Václav Šimerka to subjective interpretation of probability is discussed. The next part describes the development of probability theory in Bohemia till 1930's; the greatest stress is put on Emanuel Czuber and Bohuslav Hostinský. After a short note on probability theory teaching, the publications of Karel Rychlík and Otomar Pankraz are highlighted. They illustrate the immediate reactions to the progress in the probability theory in the thirties of the twentieth century, its reflection in university education and the elaboration of original ideas. Rychlík published the first Czech textbook (and one of the first in Europe at all) based on Kolmogorov's axiomatic theory, two papers by Pankraz contain his contribution to logical probability theory and the proposal of an axiomatisation based on the concept of conditional probability.

Back to the Main Themes

Panels

Name	Title	Theme	Language	Country
Arcavi Abraham, Barbin Evelyne (coordinator), Radford Luis, Schweiger Fritz	Mathematics of yesterday and teaching of today	2	English	Israel , France, Canada, Austria
Giacardi Livia, Gispert Hélène Kastanis Nikos, Schubring Gert (coordinator)	The emergence of mathematics as a major teaching subject in secondary schools	5	English	Italy, France, Greece, Germany

Panels
ABSTRACTS

***Theme* 2**

MATHEMATICS OF YESTERDAY AND TEACHING OF TODAY

Arcavi Abraham, Barbin Evelyne (coordinator)**, Radford Luis, Schweiger Fritz**

The purpose of this panel is to examine how history of mathematics can help us to determine what are, the essential knowledge and the procedures for today's mathematics teaching.
-History teaches - what should we learn? (**Abraham Arcavi**, Weizmann Institute, Israel),

-Perennial knowledge for mathematics teaching (**Evelyne Barbin**, Université de Nantes, France),
-Beyond anecdote and curiosity: The relevance of the historical dimension in the 21st century citizen's mathematics education (**Luis Radford**, Laurentian University, Ontario, Canada),
-“Fundamental ideas” - a link between history and contemporary mathematics (**Fritz Schweiger**, University of Saltzburg, Austria).
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***Theme* 5**

THE EMERGENCE OF MATHEMATICS AS A MAJOR TEACHING SUBJECT IN SECONDARY SCHOOLS

Livia Giacardi, Hélène Gispert, Nikos Kastanis, Gert Schubring (coordinator)

One of the most decisive characteristics in establishing public systems of education in the European countries was the introduction of mathematics – hitherto a marginal subject in the existing secondary schools – as a major teaching subject, as a constituent dimension of general education. This introduction did not occur in a homogeneous manner. Rather the forms, contents, and methodologies depended upon different cultural, social, and political contexts in the respective countries. Moreover, after the introduction, there was no steady evolution assured – in several countries, mathematics teaching suffered backlashes jeopardizing or in fact reducing its function as a major subject. The panel will confront these processes and experiences in four of the European countries:
-Mathematics becoming a constituent of general education in Prussia and Bavaria around 1810 (**Gert Schubring**, University of Bielefeld, Germany),
-Mathematics for the first time established as a major school subject in public education since the French Revolution, its decline under Restoration and eventual resurrection over the 19th century (**Hélène Gispert**, University of Orsay, France),
-Mathematics in the educational reforms of unified Italy since 1859 and ensuing conflicts (**Livia Giacardi**, University of Turin, Italy),
-Transition from private, church-organized teaching to public instruction in the new Greek state (**Nikos Kastanis**, University of Thessaloniki, Greece).

Back to the Main Themes

3-hours Workshops (based on historical and epistemological material)

(Ordered alphabetically)

Name	Title	Theme	Language	Country
Bagni Giorgio, Vicentini Caterina	History and Epistemology of Calculus and Algebra, 300 years since Leonhard Euler’s birth: cooperative learning and effectiveness of perspective teacher training	3	English	Italy
Bastos Rita, Veloso Eduardo	Episodes of the History of Geometry: their interpretation through models in dynamic geometry	2	English/French	Portugal
Becvar Jindrich, Dlab Vlastimil, Hruby Dag, Kurina Frantisek	Education of Mathematics Teachers (in Algebra and Geometry, in particular)	3	English, Czech	Canada, Czech Republic
Bessot Didier	Calculus by Augustin Fresnel (1788-1827) to improve the efficiency of parabolic reflectors	1	English	France
Bkouche Rudolf	Les aspects expérimentation des mathématiques et leur intervention dans l'enseignement	3	French	France
Boyé Anne	When they taught high school students the Chasles’ “superior geometry” (Lorsque l’on enseignait la géométrie supérieure de Chasles dès la fin du cursus secondaire).	3	English	France
Burn Robert	Towards a definition of limit	2	English	UK
Chorlay Renaud & Brin Philippe	Using historical texts in the classroom : examples in statistics and probability	2	English	France

Correia de Sá Carlos	Cinq Courbes avec Histoire: la Quadratrice, la Spirale, la Conchoïde, la Cissoïde et la Cycloïde	2	French	Portugal
Delire Jean-Michel	Kerala mathematics and astronomy; prelude to European developments.	4	English/ French	Belgium
El Idrissi Abdellah	La trigonométrie dans l'œuvre d'Al Biruni	3	French	Morocco
Estrada Maria Fernanda, Ralha Maria Elfrida	Reflexions upon a “Method for Studying Maths”, by José Monteiro da Rocha (1734-1819)	5	English	Portugal
Fried Michael, Bernard Alain	Reading and Doing Mathematics: Ancient and Modern Issues.	3	English	Israel, France
González-Martín Alejandro Santiago	Historical-epistemological dimension of the improper integral as a guide for new teaching practices	2	English & French	Spain
Guichart Jean-Paul	Viète et l'avènement du calcul littéral	3	French	France
Kurina Frantisek, Siebeneicher Christian	Algebra and Geometry in Elementary and Secondary School	1&3	English	Czech Republic, Germany
Kouteynikoff Odile	About Leonardo of Pisa's book of squares: How elementary tools can solve quite elaborate problems.	2	English	France
Lakoma Ewa	From Vitellonis' geometry to unravelling the secret of Enigma – on millennium of the Polish mathematical thought	6	English	Poland
Matos Jose Manuel	Variations in mathematical knowledge occurring in the Modern Mathematics reform movement	5	English	Portugal
Menghini Marta	The “ Eléments de Géométrie” of A. M. Legendre: an analysis of some proofs from yesterday's and today's point of view	5	English	Italy
Métin Frédéric	Adam Freitag's New Fortification	1	English	France
Michel-Pajus Anne	About Different Kinds Of Proofs Encountered Specifically In Arithmetic	3	English	France
Negrepontis Stelios, Lambrinidis Dionysios	The anthyphairetic interpretation of the mathematical method of analysis and synthesis	3	English	Greece
Polo-Blanco Irene	Alicia Boole and regular polytopes	3	English	Netherlands
Proust Christine	Histories of zeros (Histoires de zéros)	4	English	France
Racine Marie-Noelle	histoires de mathématiciennes / women mathematicians' history	4	French	France
Radford Luis	Generality and Mathematical Indeterminacy: Variables, Unknowns and Parameters, and their Symbolisation in History and in the Classroom	3	English	Canada
Thomaidis Yannis, Tzanakis Constantinos	Ancient Greek Mathematics in the Classroom	1	English	Greece

Back to the Main Themes

3-hour Workshops ABSTRACTS

(ordered by theme, 1, 2, 3, 4, 5, 6)

Theme 1

***Theme* 1**

CALCULUS BY AUGUSTIN FRESNEL (1788-1827) TO IMPROVE THE EFFICIENCY OF PARABOLIC REFLECTORS

Didier Bessot
I.R.E.M. de Basse-Normandie, Campus II – Sciences – BP 5186
Boulevard Maréchal Juin – 14032 Caen cedex, France

In 1819, François Arago asked Augustin Fresnel to take part in the Lighthouse Commission founded in 1811 which had made very little progress since then. In the first place, Fresnel began studying the existing systems and tried to improve them ; then he started exploring a new approach, even though Buffon and D'Alembert had already come up with the initial idea of compound lenses, that would later be known as Fresnel lenses, an innovation still in use today.

After having presented the background of that work, there will be a workshop whose aim will be to read through Fresnel's notes which include his calculus on the optimization of parabolic reflectors. This text, taken from the *Œuvres complètes d'Augustin Fresnel* and published from handwritten notes contained in his notebooks, is, of course, in French but mainly consists in mathematical formulas universally understood today. Comments by the presenter and the others participants may be in French and/or in English.

Technical level needed to understand the calculus : fist year of science degree.

Texts used for the workshop : extracts of *Œuvres complètes d'Augustin Fresnel*, tome 3,, Paris 1870

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***Theme* 1&3**

ALGEBRA AND GEOMETRY IN ELEMENTARY AND SECONDARY SCHOOLS

Christian Siebeneicher
Fakultät für Mathematik, Universität Bielefeld
Postfach 10 01 31, 33501 Bielefeld , Germany
Frantisek Kurina
Faculty of Education, University, Hradec Kralove Czech Republic

A Google search for "algebra" and "elementary school" provides 1.240.000 items in 0,15 seconds, and already the first ten of these show that "Algebra in Elementary School" is a well-established field of research in mathematical education. In the respective research papers the following notions containing the term "algebraic" play a central role:

algebraic thinking, algebraic understanding, algebraic reasoning, algebraic-symbolic notation, algebraic skills, algebraic proficiency, algebraic concepts, learning algebraic concepts, algebraic topics, algebraic problem solving, patterns and algebraic thinking, algebraic character of early mathematics, algebraic relations and notations.

These concepts are used in ways suggesting that their authors consider them well understood and ready to be introduced in elementary school teaching. Since teaching of elementary arithmetic often declines to formal instructions and lacks the important component of making children understand what they are doing, there is an obvious question:

Are these concepts sufficient in the education of elementary school teachers to arrive at that goal? Some carefully chosen exercises from elementary arithmetic that will be discussed with participants may help to answer this question.

The "geometry" part of the workshop will deal with three mutually related topics.

The first idea is to illustrate the shift from "Mathematics for experts" to "Mathematics for all" on three typical examples (Heron, Apollonius, Descartes).

The second idea is to show calculi as a natural method of solving some elementary problems (problem of three circles, two rectangles and two sorts of wine)

The third part will compare two languages: the symbolic language of algebra and the visual language of geometry (examples of proofs without words).

The participants of workshop will have the opportunity to work on given problems at their own pace.

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***Theme* 1**

ADAM FREITAG'S NEW FORTIFICATION

Frédéric Métin
IREM, Université de Bourgogne,
9 avenue Alain Savary, BP 47870, 21078 Dijon, France

Adam Freitag (or Fritach) was an architect for King Wladislaus IV of Poland, who sent him to Holland to study the new theories of Fortification, and bring them back to Poland. In the 1630's, he published different versions of its major work *l'architecture militaire ou la fortification nouvelle*, in which he describes the new methods invented by Marolois or Stevin. The texts show he had a perfect knowledge of the military mathematics of his time, for instance the use of new notations for decimal numbers, invented by Stevin. The book is greatly inspired by Marolois, including the plates engraved by the same artist as the one in Marolois' works, Hendrik Hondius.

The aim of the workshop is to study and understand how mathematics was used in the building of 17th century fortresses. A major part of it will be devoted to trigonometry and basic arithmetics.

To be studied in the workshop: original texts in French, translated into English (Fritach's *L'architecture militaire ou la fortification nouvelle*, Marolois' and Hondius' *Fortifications*, Simon Stevin's Works.)

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Theme **1**

ANCIENT GREEK MATHEMATICS IN THE CLASSROOM

Yannis Thomaidis
Experimental School of the University of Macedonia, Thessaloniki, Greece
Constantinos Tzanakis
Department of Education, University of Crete, Rethymnon 74100, Greece

In the first part of this workshop we shall communicate the results of a cross-curricular project that we executed, in collaboration with teachers of mathematics, language and history, in the Experimental School of University of Macedonia, Greece. The main innovation of this project was the introduction of original texts in the normal teaching of Euclidean Geometry in the 1st year of the Greek Lyceum (16 year old students). The selected texts were Euclid's *Elements* and Proclus' *Commentary on the 1st Book of the Elements*. The aim of the project was to create, among students and teachers, the atmosphere of a debate over the concept of proof and the issue of critical thinking, both of which are central in the official determination of the peadagogical role of Euclidean Geometry in the Greek Lyceum. In order to achieve the aim of the project selected propositions out of the *Elements* and Proclus' commentary on ancient critics of these propositions were being examined from different points of view: the linguistic, the historical and the mathematical. Details about the historical texts used, the worksheets completed and other didactical activities executed in the classroom will be available to the participants of the workshop.

In the second part of the workshop we shall focus on certain fundamental algebraic ideas which have been produced during a detailed study of Diophantos' *Arithmetica*, and especially of his way to cope with the issue of generality in the solution of arithmetical problems: while stating his problems in general terms, Diophantos was using particular numerical data in his solutions. The results of this study are being currently used in the preparation of didactical material for another project, which will be also executed in the same Experimental School with the aim to investigate the crossing from Arithmetic to Algebra, which takes place in the 2nd year of the Greek Gymnasium and seems to cause significant difficulties in 14 year-old students.

Back to 3-hour Workshops abstracts

Theme 2

Theme **2**

EPISODES OF THE HISTORY OF GEOMETRY: THEIR INTERPRETATION THROUGH MODELS IN DYNAMIC GEOMETRY

Rita Bastos Rita
Escola Secundária António Arroio,
Rua Coronel Ferreira do Amaral, 1900-165, Lisboa, Portugal
Eduardo Veloso
Associação de Professores de Matemática (APM)
Rua Dr. João Couto, nº 27A, 1500-236, Lisboa, Portugal

A. In this workshop the participants will use a dynamic geometry software to make geometric constructions that will interpret and model some milestone constructions proposed in the history of geometry. Four topics (and corresponding sets of historical texts – English and French versions) will be proposed for interpretation and modeling. We estimate that it will be possible to work on (only) one topic within the time allowed for the workshop. The participants, working in small groups, will have the possibility to make a choice of the topic to work on. At the last period of the workshop, participants will share their work and a general discussion of the meaning and usefulness of this kind of work will follow. Proposed topics and texts are the following:

1. Piero della Francesca (c. 1410-1492).
   - "On the perspective plane to construct [the image] of a given square"; from the book *De Prospectiva Pingendi* (On perspective for painting), before 1482.
2. Albrecht Dürer (1471-1528) and Germinal Pierre Dandelin (1794-1847).
   - Conic sections by double projection and Dandelin spheres
     - A. Dürer, text from the book *Underweysung der Messung mit dem Zirckel und Richtscheyt...* (Instruction in Measurements with Compass and Ruler in Lines, Planes and Solid Bodies), Nuremberg, 1525.
     - G. Dandelin, text from the article *Mémoire sur l'hyperboloide de révolution, et sur les hexagones de Pascal et de M. Brianchon*, Nouveaux Mémoires de l'Académie Royale des Sciences et des Belles-Lettres de Bruxelles, Classe de Sciences, 1826
3. Gilles Personne de Roberval (1602-1675) and René Descartes (1596-1650).
   - The tangent to the cycloid
     - G. Roberval, text from the article *Observations sur la composition des mouvements et sur le moyen de trouver les touchantes des lignes courbes*, Recueil de l'Académie, tome VI, 1693.

• R. Descartes, text from a letter to Père Mersenne (1638), Œuvres, t. II.

4. Gaspard Monge (1746-1818)

• Construction of the planes tangent to a sphere and containing a given line

• text from the book *Géométrie Descriptive*, 1799.

B. Complete guidance and hints on the use of the software will be given as handouts, computer files and direct help. The program *Geometer's Sketchpad*, version 4, will be used in this workshop, but other dynamic geometry software (like Cabri) could be used later to solve the same questions.

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***Theme* 2**

TOWARDS A DEFINITION OF LIMIT

Robert Burn
Professor Emeritus Agder University College, Kristiansand, Norway
Previously Reader in Mathematics Education, Exeter University,
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TOWARDS A DEFINITION OF LIMIT

Archimedes' quadratures of the spiral and parabola and their consequences in the 17th century in the work of Fermat, Gregory of St Vincent and Wallis.

<u>Expected level of audience</u>

Any teacher who may introduce and define the notion of limit, at high school or university, and who may be interested in integration without calculus.

<u>Workshop 1</u>

Euclid V. Def. 4

Quadrature of the spiral.

Encapsulation of Archimedes' argument in a 'vice' [$-e < A < e$ for all positive e].

Application of the spiral method by Fermat and elsewhere.

<u>Workshop 2</u>

Euclid X.1

Quadrature of the parabola,

Use of finite geometric progressions.

Refining the 'vice'.

Generalisation of Euclid X.1 by Gregory of St Vincent.

<u>Workshop 3</u>

Infinite geometric progressions in Gregory of St Vincent.

Their application by Fermat.

Extending the 'vice' to define a limit.

Wallis' infinite product as a limit.

The workshop will be structured as problem-solving experiences.

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***Theme* 2**

USING HISTORICAL TEXTS IN THE CLASSROOM: EXAMPLES IN STATISTICS AND PROBABILITY

Renaud Chorlay, Philippe Brin
IREM Paris 7, 175-179 rue du Chevaleret, Plateau E, 75013 Paris, France

Founded in the early 1980's , the M:A.T.H.* group works on the introduction of a historical perspective, both in the classroom and in the training of teachers. The work centres on the use of *genuine* historical texts. We would like to present a few texts which we have used in the classroom, at high school level (5th – 7th form), on topics pertaining to statistics (mean, median, life expectancy) or elementary probability theory (games of dice). We may read excerpts from Galileo, Pascal, Fermat, the Huygens brothers and Leibniz.

* M:A.T.H. = Mathématiques, Approche par les Textes Historiques

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***Theme* 2**

CINQ COURBES AVEC HISTOIRE: LA QUADRATRICE, LA SPIRALE, LA CONCHOÏDE, LA CISSOÏDE ET LA CYCLOÏDE

Carlos Correia de Sá
Departamento de Matemática Pura da Faculdade de Ciências da Universidade do Porto
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L'atelier comportera l'étude de cinq courbes historiquement importantes (la Quadratrice d'Hippias et Dinostrate, la Spirale d'Archimède, la Conchoïde de Nicomède, la Cissoïde de Dioclès et la Cycloïde de Roberval) et la résolution de plusieurs problèmes géométriques au moyen de ces courbes. Les activités de l'atelier seront soutenues par la lecture de textes historiques au contenu mathématique. Je disponibiliserai une colléction abondante de textes, dont on choisira ceux qui seront lus en atelier, selon les goûts et les préférences des assistants.

Pour les quatre premières courbes, je fournirai des textes d'Archimède, de Dioclès, de Pappus, de Proclus et d'Eutocius. Nous verrons les problèmes géométriques auxquels ces textes sont liés et comment ces courbes permettent de les résoudre.

Nous prendrons contact avec les propriétés de la cycloïde à travers un texte de Roberval concernant le tracé des tangentes. Puis je proposerai de voir comment une cycloïde nous permet, elle aussi, de quadrer un cercle et de trissecter un angle.

Les textes utilisés seront les suivants.

Pour la Quadratrice:

-Pappus d'Alexandrie, *Colléction Mathématiques* IV, 30.

-Proclus de Lycie, *Commentaires sur le premier livre des Éléments d'Euclide* (commentaire à la proposition I, 9 des *Éléments* d'Euclide).

Pour la Spirale:

-Archimède, *Des Spirales*, définitions, propositions 12, 14 et (18).

Pour la Conchoïde:

-Pappus d'Alexandrie, *Colléction Mathématiques* IV, 32.

-Eutocius d'Ascalon, *Commentaire sur le traité de la Sphère et du Cylindre* II (commentaire sur la synthèse de la proposition 1 – solution à la manière de Nicomède dans son livre sur les Lignes Conchoïdes).

Pour la Cissoïde:

-Dioclès, *Les Miroirs Ardents*, propositions 11, 12, 13, 14 et 15.

-Eutocius d'Ascalon, *Commentaire sur le traité de la Sphère et du Cylindre* II (commentaire sur la synthèse de la proposition 1 – solution à la manière de Dioclès dans son livre sur les Miroirs Ardents).

Pour la Cycloïde:

-Gilles Personne de Roberval, *Observations sur la Composition des Mouvements et sur les Moyens de trouver les Tangentes aux Lignes Courbes* (problème 1 – Onzième exemple de la Roulette ou Trochoïde de M. de Roberval).

J'utilise les suivantes traductions en français des textes grecs, latins et arabes:

R. Rashed 2002: *Les Catoptriciens Grecs*, tome 1, *Les miroirs ardents*. Paris.

P. Ver Eecke: *Proclus de Lycie – Les commentaires sur le premier livre des Éléments d'Euclide.* Bruges, 1948.

P. Ver Eecke: *Les Oeuvres Complètes d'Archimède, suivies des commentaires d'Eutocius d'Ascalon*, 2 volumes, Paris, 1960.

P. Ver Eecke: *Pappus d'Alexandrie – La Collection Mathématique*, 2 volumes, Paris, 1982.

J'espère pouvoir trouver de bonnes traductions en anglais de tous les textes, pour que l'atelier puisse être bilingue. Cela ne devra pas poser de problème pour Archimède (avec Eutocius), Pappus et Proclus, mais il me faut encore trouver des traductions anglaises de Dioclès et de Roberval.

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***Theme* 2**

HISTORICAL-EPISTEMOLOGICAL DIMENSION OF THE IMPROPER INTEGRAL AS A GUIDE FOR NEW TEACHING PRACTICES

Alejandro Santiago González-Martín

Département de Didactique, Faculté de Sciences de l'Éducation, Université de Montréal

Succursale Centre-Ville, Montréal (Québec) – H3C 3J7, Canada

The proposed workshop is based both on historical and didactical materials and it was originated as a part of my PhD Thesis, regarding the teaching and learning of the concept of improper integral. I will present some of the results of my experimentation and the innovative teaching method (based on a historical-epistemological analysis) we designed.

To study the teaching-learning of the improper integral, an analysis of how university students learn this concept was developed to later design a Didactic Engineering to improve this learning and the traditional teaching practices. Our design was founded on the classical analysis of three dimensions prior to de design of a Didactic Engineering: epistemological, cognitive and didactical.

The improper integral is generally introduced at University level as a simple extension of the Riemann integral when the interval of integration is not bounded. Teaching focuses the learning of convergence criteria and the resolution of prototypical examples and little time is dedicated to the interpretation of what is happening. This way, many misconceptions remain (the idea that a figure will enclose finite area only if it is closed and bounded, the idea that if an integral is convergent then the integrand must tend to zero, the idea that $\int_a^{+\infty} f(x)dx$ and $\sum_{n=1}^{+\infty} f(n)$ have the same character, …).

Our analysis of the evolution of this concept through History shows that although it appeared as a natural extension of the calculation of areas, its origins are linked to the use of the graphic register and of series. Moreover, many results were accumulated before a general theory was developed and some paradoxes were subject of discussion between the philosopher Hobbes and the mathematician Wallis.

We developed our Didactic Engineering in the University of La Laguna (Spain) inversing the traditional teaching sequence, so we studied many particular cases before a theory was established, to develop an intuition in our students, and we also gave the graphic register a big importance. We also used actively the results of series to construct many counterexamples to the typical misconceptions. The students were responsible of many results, so these results were the product of their own activity.

In this workshop we intend the following:

1.- Analyse how improper integrals are usually taught at University and give some clues to explain this.

2.- Analyse some texts (particularly Fermat's) to see how and with which motivation these integrals appeared.

3.- Give an overview of misconceptions that students develop with the study of traditional examples.

4.- Try to give some ideas and activities to use the historical motivations to teach this concept and analyse their efficiency.

The workshop is intended to be interactive and open to discussion and contributions of the participants, adding new activities and situations to the experience that was already developed in Spain.

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***Theme* 2**

ABOUT LEONARDO OF PISA'S BOOK OF SQUARES: HOW ELEMENTARY TOOLS CAN SOLVE QUITE ELABORATE PROBLEMS

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Leonardo of Pisa (1170-1240), known as Fibonacci, had an opportunity to learn the Indian art of calculating, as a teenager, when staying in Algeria with his father, and as a young man, while sailing along from one Arabic Mediterranean country to another, for his own business trips. It seems he returned to Pisa when about 30.

King of Sicily Frederic II (1194-1250), who was the grand son of red-bearded Frederic I, was raised Germanic emperor in 1212 and enjoyed being surrounded by a circle of fine scholars. Holding court in Pisa about 1225, the sovereign wanted to meet Fibonacci. Master Johannes of Palermo then took the opportunity to submit to him the question of finding a square and a number such that both the sum of the two and their difference are squares too. Fibonacci's Book of squares (Pisa, 1225) is devoted to his genuine and smart solution for this henceforth well-known question.

It is based on a very fresh consideration of the Euclidean properties and a very keen intuition of what we now call "the number theory". Fibonacci offers material to his readers in a systematic way, orders things from the easiest to the more difficult and gives a proof for any result he appeals to, which proof is to be seen on a Euclidean line segment figure. He knows the property of squares as sums of odd numbers. He also knows the rules for the ordered sums of the squares of running from 1 consecutive or odd numbers.

The aim of the workshop is to read and discuss the former questions. We hope you will enjoy it and will be persuaded that your pupils might enjoy it too. According to their school level, they will be able to understand the results either through an inductive reasoning or through a quite formal proof. And for further ambition, Fibonacci's treatise provides material to reflect

- on limits of natural language and on the way complex calculations are carried out more easily with symbols
- on the efficiency of elementary tools to solve quite elaborate problems within the specific field of arithmetic
- on the way ancient texts can bring historical information about their time, author and subject, and above all throw light on questions of the upper kind.

Fibonacci's treatise is available both in French and in English:

[1] Ver Eecke P., Léonard de Pise. Le livre des nombres carrés, Paris, Librairie scientifique et technique Albert Blanchard, 1952.

[2] Sigler L. E., Leonardo Pisano Fibonacci's book of squares, Boston, Academic Press, 1987.

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Back to 3-hour Workshops abstracts

Theme 3

Theme 3

HISTORY AND EPISTEMOLOGY OF CALCULUS AND ALGEBRA, 300 YEARS SINCE LEONHARD EULER'S BIRTH: COOPERATIVE LEARNING AND EFFECTIVENESS OF PERSPECTIVE TEACHER TRAINING

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In this workshop we shall propose an exchange of ideas about the role of history and epistemology of mathematics in perspective teachers training. We shall make reference to some historical references, in order to celebrate third centenary of Leonhard Euler's birth (1707).
Both the authors have been in the situation of giving a 20 hours teachers training course for the "Scuola di Specializzazione per l'Insegnamento Secondario" at the University of Udine, and they would move from their own experience. They will start by posing some general questions about how to organise a course on history and epistemology of mathematics having as principal aim the idea of overcoming the usual gap between theory and practice in mathematics education. They will use the cooperative learning techniques in order to allow participants to explore the subject and to catch some shared (even if necessary partial) conclusions.
Afterwards, some pages from Euler's treatise entitled *Vollstandige Anleitung zur Algebra* (proposed both in the English 1828 edition, and in the French 1807 edition) about Diophantine equations will be considered as starting material to plan a lesson for perspective teachers. Our aim is to help participants to practice with the organisation of a didactical unit based on historical references. Of course in this second part participants are supposed to be consistent with the conclusions reached in the first part.
Finally, another question will be submitted to participants: we shall discuss the opportunity to provide a socio-cultural analysis of different proofs of a "same theorem" produced in different times and situations. The case analysed will concern the infinity of prime numbers, namely Euclid's, Kummer's and Euler's proofs.
In particular, we shall make reference to the following texts (we are planning to prepare a PDF version of the original texts in a website, to be downloaded by the participants):

- Euler, L. (1796). *Introduction a l'Analyse Infinitésimale*, Barrois, Paris 1796, first edition in French, vol. I: pp. 208-213.
- Euler, L. (1828). *Elements of Algebra, with the notes of M. Bernoulli, &c. and the additions of M. de La Grange*. Longman, Rees, Orme and Co., London, II, pp. 299-312.
- Kummer E.E. (1878). Neuer elementarer Beweis, dass die Anzahl aller Primzahlen eine unendliche ist. *Monatsber. Akad. D. Wiss. Berlin*, 1878/9, 777-778.
- Tartaglia, N. (1959). *Euclide Megarense acutissimo philosopho*. Bariletto, Venice, p. 171 (original text; English version in: Heath T.L. (1952). The thirteen books of Euclid's Elements. In Maynard Hutchins R., ed., *Great Books of the Western World* 11. W. Benton-Encyclopaedia Britannica, Chicago-London-Toronto 1-402. p. 184).

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Theme 3

EDUCATION OF MATHEMATICS TEACHERS (IN ALGEBRA AND GEOMETRY, IN PARTICULAR)

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This Workshop will address mainly the education of teachers of Mathematics at all school levels. This circumstance is underlined by the fact that the panel is represented by teachers from all levels of Mathematics education. The Workshop will relate closely to the earlier Workshop "Knowing, Teaching and Learning Algebra " and will discussed with participants some of the points raised there.
For the benefits of the local teachers, the discussions will be run simultaneously in English and Czech. Discussions of very specific questions and situations arising in teaching of Mathematics classes, presented either by the panel or introduced by participants, will be the main objective of this Workshop. The ultimate aim will be

to bring about suggestions for improving preparations of the teachers of Mathematics for their task This will encompass a curriculum directory specifying a carefully chosen list of topics, reflecting history of Mathematics and underlying its Unity, as well as suggestions for conducting some of the classes with a material reflecting the above points.

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***Theme* 3**

LES ASPECTS EXPERIMENTATION DES MATHEMATIQUES ET LEUR INTERVENTION DANS L'ENSEIGNEMENT

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Emile Borel proposait en 1904 l'installation de laboratoires de mathématiques dans les établissements d'enseignement secondaire. Cette question des laboratoires de mathématiques renvoie à la part expérimentale de l'activité mathématique et c'est sur ce point que nous nous proposons d'intervenir.
La question de la part expérimentale dans l'activité mathématique porte peut-être plus sur le terme "expérimental" que sur le terme "mathématiques" et nous l'aborderons d'un point de vue gonséthien en revenant sur les trois aspects, l'intuitif, l'expérimental et le théorique, que développe Gonseth en ce qui concerne la géométrie.
Cela nous conduira à expliciter comment cet aspect expérimental a sa place dans l'enseignement des mathématiques à travers divers exemples pris dans divers chapitre des mathématiques enseignées dans l'enseignement élémentaire et secondaire. A titre d'exemples :

- *arithmétique*, d'une part la calcul, l'aspect expérimental apparaissant avec la main (compter sur ces doigts), le boulier et le calcul écrit.
- *géométrie*, nous insisteronst sur les instruments de mesure des grandeurs géométriques et sur les instruments de dessin dont la règle et le compas représentent les prototypes. Les constructions géométriques ont conduit à définir et construire des instruments géométriques divers parmi lesquels les systèmes articulés ont joué un rôle important.
- *analyse*, nous insisterons sur la notion de fonction définie comme une loi qui associe à une valeur de la variable indépendante (par exemple le temps ou la température) la valeur d'une grandeur dépendant de cette variable et la signification d'une représentation graphique comme représentation géométrique. Dans ce cadre la construction de la représentation graphique peut être considérée comme une activité expérimentale. Nous pourrons compléter ces remarques par une étude qualitative des équations différentielles.

Nous n'avons pas encore parlé de l'outil informatique. Disons d'abord que l'emploi précoce dans l'enseignement des calculatrices et plus généralement de l'ordinateur masque l'aspect expérimental.
D'abord parce que l'expérimental et le théorique sont liés et qu'il ne saurait être question de présenter une activité expérimentale coupée de toute approche théorique.
Ensuite, pour que l'usage de l'informatique ne se réduise pas à quelques recettes magiques, il importe que les constructions effectuées sur une machine s'appuient sur une pratique mathématique déjà consistante.
Cela nous conduira à parler du rapport entre l'analogique et le numérique.
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***Theme* 3**

WHEN THEY TAUGHT HIGH SCHOOL STUDENTS THE CHASLES' "SUPERIOR GEOMETRY" (LORSQUE L'ON ENSEIGNAIT LA GEOMETRIE SUPERIEURE DE CHASLES DES LA FIN DU CURSUS SECONDAIRE)

Anne Boyé
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The Chasles' geometry, that we call in France "géométrie supérieure", has been taught in high schools, from the end of XIXth century up to the 1960's, at least in France. Was it a "good" mathematical education ? For what reasons this teaching has been given up ?

– obsolete ?
– did not fit the "new students" ?
– they put other teachings in place of it ?

During the workshop, extracts of English and French text and exercise books from different periods will be studied, so that, if necessary, you will be introduced to this geometry and try to answer those questions.

La "géométrie supérieure" de Michel Ckasles a été enseignée dans les lycées, dès la fin du XIX° siécle et ce jusque dans les années 1960, du moins en France. Etait-ce une" bonne" formation mathématique ? Pour quelles raisons cet enseignement a-t-il été abandonné ?

-Désuet ?
-Mal adapté aux “nouveax élèves”
-D’autres eneigenemnts l’ont-ils remplacé ?
Dans l’atelier seront étudiés des extraits de manuels d’enseignement français et anglais, de différentes époques, pour s’initier éventuellement à cette géométrie, et essayer de répondre à ces questions.
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***Theme* 3**

LA TRIGONOMETRIE DANS L’ŒUVRE D’AL BIRUNI

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Al Biruni est un mathématicien du onzième siècle. Il s’est intéressé à plusieurs sciences et ses travaux en mathématiques sont nombreux. Notre objectif en fait est de présenter une histoire de la trigonométrie dans la civilisation arabo-musulmane, histoire articulée autour des travaux d’Al Biruni. Nous ne pouvons prétendre présenter toutes les œuvres d’Al-Biruni ; nous nous référerons particulièrement à deux traités :
1- Al Qanun Al Masoudi, notamment au troisième chapitre. Dans ce chapitre, al-Biruni expose sa méthode pour calculer une table de sinus (cordes). Pour calculer certains sinus particuliers, il fait usage de divers artifices et expose une méthode d’interpolation quadratique ingénieuse. Une particularité d’Al Biruni est qu’il situe souvent ses propres travaux par rapport à ces ancêtres (Grecs et Hindous) et à ces contemporains.
2- L’extraction des cordes de cercles (istikhraj al-aoutar fi ad’daira) : ce qui caractérise ce traité est sans doute le fait qu’il s’appuie sur une propriété géométrique fort intéressante et relativement oubliée : le théorème de la corde brisée. Al-Biruni en donne une vingtaine de démonstrations ainsi que quelques corollaires. Il fait usage de cette propriété dans la résolution de problèmes relatifs à la trigonométrie, à la géométrie, à l’astronomie et à l’algèbre.
Dans cet atelier, nous évoquerons les éléments suivants :
- Eléments d’histoire de la trigonométrie Arabo-musulmane ;
- Les formules et propriétés trigonométriques utilisées par Al-Biruni ;
- Le théorème de la corde brisée, ces démonstrations et ses corollaires ;
- Le calcul de sinus 40° (trois méthodes) ;
- Le calcul de sinus 1° (trisection de l’angle) ;
- Le procédé d’interpolation quadratique ;
- Quelques applications du théorème de la corde brisée ;
- Quelques caractéristiques de l’activité mathématique chez Al-Biruni.

Les documents de travail consisteront en des extraits (originaux ou traduits) qui orienteront les activités et supporteront les explications. L’atelier laissera de la place à la discussion d’applications pédagogiques éventuelles.
L’atelier s’adresse à tout enseignant et à tout formateur d’enseignant de mathématiques.
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***Theme* 3**

REFLEXIONS UPON A “METHOD FOR STUDYING MATHS”, BY JOSÉ MONTEIRO DA ROCHA (1734-1819)

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This is a workshop designed to acquaint the participants with a broad spectrum of mathematical ideas and viewpoints presented in a manuscript written, in the XVIIIth century, by the Portuguese priest (and very influential scholar during the reform of the University of Coimbra) José Monteiro da Rocha, as an introduction to a mathematics course.
Based on both Monteiro da Rocha’s life and on his own comments on the teaching and learning of mathematics in Portugal, we have strong reasons to date this unpublished manuscript around 1760(s). At that time, the study of maths as well as the study of other areas, was at such a low level that some scholars were quite concerned with the learning of contents and the methods of learning; and as a consequence we can find some important suggestions for dealing with the problem such was the case of Luís António de Verney’s “Verdadeiro Método de Estudar”, published in 1750 which acquired a quite large publicity and that is, even nowadays, a source of research. Less known that Verney’s work, and other similar texts produced all over Europe at that time, appears to be the above-cited manuscript by Monteiro da Rocha. He himself, echoes his concern with the teaching and learning of mathematics, by reporting on the lack of teachers of mathematics and intending to do something for, according to his own words, “(to) wake up his nationals from the lethargic state (they were living in)”; in particular he suggests “writing in vulgar language (Portuguese instead of Latin)” for spreading mathematical

knowledge among Portuguese citizens and he stimulates the national pride by reporting on the example of "(our ancestors') glorious maritime discoveries".
In the present workshop we will:
i) Present a brief summary on the life and works of José Monteiro da Rocha, referring, in particular, to his activity in designing and writing the rules for the big reform of the University of Coimbra (1772) where, for the firs time, a Faculty of Mathematics was created in Portugal; we will as well refer to his activity as a professor of the same university and his relationships with the academics of Real Academia das Ciências de Lisboa; we will also report on the international spreading of some of his works on Astronomy;
ii) Refer to the ideas/models presented in other textbooks of Monteiro da Rocha's times;
iii) Propose the analysis of some specific parts of the above cited introduction, by reading them from an English version of the original manuscript distributed to the participants;
iv) Lead a follow-up discussion/reflection on
- the actuality of Rocha's ideas, for example on his understanding of what is mathematical knowledge;
- the development/history of the models for presenting/introducing mathematics textbooks;
- the historical-didactical facts that we can learn/apply, at the present time where we are also acknowledging a decreasing interest in studying mathematics.
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***Theme* 3**

READING AND DOING MATHEMATICS: ANCIENT AND MODERN ISSUES

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Partly through the influence of constructivist theories, research in mathematics education has for many years now tended away from predetermined mathematical material approached in predetermined ways. Argumentation, communication, investigative activities, and student productions—matters which emphasize students' own part in acquiring mathematical understanding—have accordingly become dominant themes in teaching and research. This tendency, on the face of it, seems at odds with historians' disciplined readings of mathematical texts, their distancing themselves from their own modern preconceptions, and their fixed desire to read texts as the authors wrote them. And yet, historically, in the humanist educational tradition, the reading of texts appears to have been more in the spirit of those themes of mathematics education to which we referred just now. One simple reason for this is that ancient education, with his emphasis on rhetoric, aimed at developing an ability to *produce* texts or discourses. Ironically, then, by reading historical texts with these current mathematics education tendencies in mind, we are, as Collingwood might put it, reenacting the historical context of the reading of these texts; we are, in this way, truly engaging in an historical study while developing our own mathematical sensibilities.
The purpose of this workshop, then, will be to demonstrate how historical texts may be read in the light of this humanist tradition; by so doing, we hope to show how teachers may be made aware of an earlier model for recent approaches in mathematics education and thus provide a historic framework in which modern issues about mathematical investigation and activity may be discussed. We also hope the workshop will show the possibility of interesting and non trivial collaborations with teachers from traditionally 'humanistic' *cursus* (language, history, philosophy). All of the authors we shall discuss will be from the classical period and will include Plato, Isocrates, Euclid, Vitruvius, Pappus, and Proclus. Some of these authors will be used to provide a general sense of context; the active part of the workshop, however, will draw chiefly from Euclid's *Elements* and *Data* and Proclus's *Commentary on the First Book of Euclid's Elements*. Finally, we shall want to discuss, as a summing up, how the methods of the humanistic tradition were inseparable from its content. We believe that this historical question of content and method may also be a source of insight for understanding those currents in mathematics education mentioned above.
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***Theme* 3**

VIETE ET L'AVENEMENT DU CALCUL LITTERAL

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1) L'apport de Viète
2) La résolution des équations : changement de point de vue.
Étude d'un exemple chez Al Khwarismi et chez Viète.

Le rôle des identités remarquables.
Exemples d'utilisation en classe (collège-lycée)
3) La résolution des problèmes de construction par l'algèbre.
Étude d'un exemple chez Al Khwarismi et chez Bezout.
Mise en équation des problèmes.
Exemples d'utilisation en classe (collège-lycée)
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***Theme* 3**

ABOUT DIFFERENT KINDS OF PROOFS ENCOUNTERED SPECIFICALLY IN ARITHMETIC

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One of the interesting aspects of arithmetic is that, without needing a large theoretical arsenal, mathematical proofs can be constructed which are supported by reasoning of a certain subtlety, playing with the notions of infinity and the absurd, and hence obtaining non-trivial results. This reasoning, because it relates to the integers, is easily accessible intuitively. This gives a specific formative character to arithmetic in the apprenticeship of proof.
The history of mathematics offers us a large choice of proofs, some more formal, some less, some further from intuition, some closer. Moreover, we can refer to commentaries by mathematicians on the elegance or the rigour of certain proofs.
The corpus of texts we have chosen for reading revolve around the "Fermat's Little Theorem" (part of the final programme in secondary school). The basic theoretical baggage is then limited to a single property which appears in different forms according to point of view and to the context (Euclid's Lemma, Gauss' Theorem, The Fundamental Theorem of Arithmetic). The essential core of methods of proof also manifests itself in different forms (infinite descent, the principle of recursion, the use of the smallest integer in a set of integers).
We shall set out the principal points of our analysis, supported by the reading of original excerpts. A detailed article, including all the source texts, will be available on the IREM Paris 7 site.
Excerpts from: Fermat, Euler, Prestet, Legendre, Gauss, Tannery, etc.
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***Theme* 3**

THE ANTHYPHAIRETIC INTERPRETATION OF THE MATHEMATICAL METHOD OF ANALYSIS AND SYNTHESIS

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Analysis and synthesis is commonly perceived as a mathematical method of deduction; its most authoritative description is given by the Greek mathematician Pappus in his work "Mathematical Collection" (7.634.2−635.18). This method consists of two stages. In the first stage (analysis) there is a deductive procedure consisting of a finite number of steps, starting from the proposition to be proved and ending when a statement already known is reached. In the second stage (synthesis) the desired theorem or construction is established by reversing the steps of the analysis stage (assuming that such a reversal is mathematically possible). A description of the mathematical method and of the connected problems of interpretation is presented in [5], [6], [7], [8], [11], [12], [24], [29].
It is quite interesting that neo-Platonists (as Iamblichus in "De communi mathematica scientia", 20.1−31 and Proclus in "Commentary on the first Book of Euclid's Elements" 42.9−43.21, 57.14−58.19) correlate the mathematical method of analysis and synthesis (a) with the philosophical method of Plato's dialectic "diairesis and synagoge" ('division and collection'); and (b) with the celebrated passage of the "Divided Line" ("Republic" 509d.6−511e.5), where Plato exposes his doctrine about the ontological levels and the knowledge of beings, using a geometric metaphor; according to it, hypotheses are correctly used by dialecticians to grasp the first principle (described as "unhypotheton"), a step described as an upward movement, while on the contrary the geometers, according to Plato, use hypotheses incorrectly, and are delegated to the downward movement.
Various modern commentators (as T. Heath in [7], F.M. Cornford in [3], R. Robinson in [25], I. Mueller in [13], I. Zhmud in [28]), following the neo-Platonists, in their attempt to understand Plato's dialectics in the Republic, have correlated the "upward movement" of Plato's dialectic with a philosophical analogue of "analysis" and the "downward movement" with a philosophical analogue of "synthesis". Others, including H. Cherniss in [2], oppose this neo-Platonic interpretation.
These modern interpretations are certainly not totally wrong, but they are based on some fundamental misconceptions mostly at the philosophical level and, as a result, are not sufficiently penetrating as to clarify, in a satisfying manner, the correlation of the mathematical method of analysis and synthesis with Plato's theory of

Forms and Ideas. What is lacking is a deep and essential interconnection of Plato's dialectics with ancient Greek mathematics. We propose that the key concept is the notion of anthyphairesis (corresponding to the modern notion of continued fraction), as developed in the studies originated by S. Negrepontis since 1996, and detailed in a number of publications ([14], [15], [16], [17],[18],[19],[20]), and M.Sc. theses ([1], [9], [10], [21], [22], [23]) at the Department of Mathematics of Athens University under his supervision.

The anthyphairetic interpretation of analysis and synthesis rests on the following two key steps:

(A) The anthyphairetic interpretation of the platonic method of division and collection, as described in the dialogues "Theaetetus", "Sophist", "Statesman", Phaedrus, "Parmenides", according to which division is the philosophic analogue of anthyphairetic division and collection is the philosophic analogue of the logos criterion, as described e.g. in the works of van der Waerden [27] and Fowler [4], by which periodicity and hence self-similarity, and collection of all parts into one is achieved. This process is clearly seen in the divisions of the Angler and the Sophist in the Sophistes, in the divisions of the art of Weaving and the Statesman in the Statesman, and in the description of the One in the Second hypothesis of the Parmenides. This step has been described in detail in the works by one of the authors (S.N.) cited above.

(B) Hypotheses play a central role in the Republic (cf. passage 533a.8−534d.1, in addition to the one given above), the Phaedo and the Meno, but are frequently mentioned in the Parmenides, and in the division steps in the Sophistes and the Statesman. We provide detailed documentation that the hypotheses in Plato appear always as opposing pairs and their meaning in the dialogues Republic, Phaedo and Meno essentially coincides with the meaning of division in the dialogues Sophistes, Statesman, and Phaedrus (and hence with the anthyphairetic division by (A)); and that the 'the anhupotheton' of the Republic, the 'hikanon' of the Phaedo', and the 'logos' (or logismos') of the Meno essentially coincides with the collection in the dialogues Sophistes, Statesman, and Phaedrus (and hence with the anthyphairetic logos criterion, by (A)). Toward this interpretation some comments by Proclus [exact references] are especially valuable; in particular his description of the way Pythagoreans "provide logos" to the hypothesis of the "three kind of angles" (one of the mathematical hypotheses mentioned in the "Divided Line" passage). This step is a joint work by the two authors, forming the basis of a doctoral dissertation by D.L. under the supervision of S.N.

According to the "anthyphairetic" interpretation (employing (A) and (B) above), the "upward movement" of dialectic and the 'analysis' correspond to a philosophical analogue of an anthyphairetic division process whose parts arising from the division are the hypotheses, coming always in pairs of opposites; this division process aims at the unhypotheton, i.e. at collection, the philosophical analogue of the "logos criterion" of the mathematical anthyphairesis, which when found provides complete knowledge of the Idea or Form, as in the anthyphairetic situation of the 'commensurable only in power' pair of lines, and also as the 'analysis' results after a finite number of steps in something known. The geometers do not aim at logos but derive one set of hypotheses from another, only dividing and not collecting. 'Synthesis' now deals only with forms (as described in the Republic…) and, by periodicity, returns to and establishes the knowledge of the original entity on which analysis and synthesis is applied.

In conclusion, the anthyphairetic interpretation of analysis and synthesis provides a mathematical interpretation of a mathematical method, but one that depends wholly on the mathematical interpretation of the Platonic philosophy. The whole argument reveals the close, indeed inseparable, interrelation of ancient Greek philosophy with mathematics.

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***Theme* 3**

ALICIA BOOLE AND REGULAR POLYTOPES

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The five Platonic Solids (the tetrahedron, the cube, the octahedron, the dodecahedron and the icosahedron) can be generalized to four or more dimensions. The solids in four dimensions are called "regular polytopes". One way a three-dimensional viewer can comprehend the structure of a four-dimensional polytope is through calculating its sections. We will begin by studying the sections of some of the three dimensional solids, and generalize the idea to the fourth dimensional ones. For that purpose, Alicia Boole's method on calculating the sections of the regular polytopes will be described. We will propose to calculate some of the simplest sections as an exercise, and provide enough material to build models of them. A brief biography of Alicia Boole's life will also be given.

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***Theme* 3**

GENERALITY AND MATHEMATICAL INDETERMINACY: VARIABLES, UNKNOWNS AND PARAMETERS, AND THEIR SYMBOLISATION IN HISTORY AND IN THE CLASSROOM

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Because mathematical objects are general, *indeterminacy* –understood as something precise and yet not particular– is one of the chief characteristics of mathematical activity. Variables, unknowns and parameters are interrelated central features of indeterminacy. From the viewpoint of the historical conceptual development of mathematics, using signs to distinguish and designate them has been carried out through a lengthy process. The goal of this workshop is twofold. First, we will read and discuss some original sources (Hypsikles, Diophantus, Descartes and others) in order to see how variables, unknowns, and parameters were instrumental conveyers of indeterminacy in the shaping of mathematical generality. Second, we will analyze some videotaped passages of High School students. The expected outcome of the workshop is a better understanding of (1) the role that symbols (and semiotics) play in shaping indeterminacy and mathematical generality, and (2) the difficulties that students encounter in dealing with the general.

Back to 3-hour Workshops abstracts

Theme 4

***Theme* 4**

KERALA MATHEMATICS AND ASTRONOMY; PRELUDE TO EUROPEAN DEVELOPMENTS

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Je voudrais, dans un atelier de 3h., aborder les contributions les plus remarquables, dans le domaine de la trigonométrie, des mathématiciens-astronomes Kéralais des XIV-XVIè siècles (comme Niilaka.n.tha Somayaji, Daamodara, son maître, ou même le père de ce dernier, Parame'svara, qui est connu pour avoir inventé le Drgg.nita, une méthode particulière de calcul astronomique, mais aussi Jye.s.thadeva, auteur de la fameuse Yuktibhaa.saa, ou 'Sa'nkara et Naaraaya.na, auteurs de commentaires importants d'oeuvres, parfois non kéralaises mais fort anciennes, comme la Liilaavatii ou même l'Aaryabha.tiiya, etc) . Bien sûr, ce sont les Indiens qui ont inventé le concept qui a mené à notre sinus (demi-corde en sanskrit), mais au Kerala, on a plutôt approfondi les méthodes de calcul des tables, utiles à l'astronomie, de ces sinus, ainsi que développé, sur une base géométrique, des séries permettant de trouver de très bonnes approximations, par exemple, de la circonférence d'un cercle de rayon donné, des nombres trigonométriques d'arcs non tabulés, de l'arc correspondant à un nombre trigonométrique donné, etc. Comme j'ai accès aux textes, souvent en sanskrit (pas la Yuktibhaa.saa, qui est à l'origine en malayalam, mais il existe une mauvaise traduction sanskrite et une bonne traduction anglaise), j'illustrerai ces notions par des extraits, choisis pour leur caractère compréhensible, de manière à amener les participants à les traduire en langage mathématique d'aujourd'hui tout en comprenant que ces résultats étaient à l'origine complètement exprimés par des mots. Bien entendu, je placerai tout cela dans le contexte particulier du Kerala, où les mathématiciens-astronomes, jeunes et vieux, vivaient collectivement dans des illam (mot désignant la maison, la résidence), grâce au patronage du roi auquel ils rendaient les services habituels à leur savoir (astrologie, établissement de calendriers, etc.), et j'essayerai finalement de mettre leurs résultats en contraste avec ceux des mathématiciens européens du XVIIè siècle, ce qui suscitera certainement de profondes questions. L'atelier sera en anglais et français.

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***Theme* 4**

HISTORIES OF ZEROS
(HISTOIRES DE ZEROS)

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In this workshop, I will show the range of contexts in which what we now call zero appears. To demonstrate the problems arising from this multifaceted invention, I will focus on examples from Mesopotamia. In cuneiform texts, the scholarly writing of numbers is based on a positional system, but without zeros in terminal positions. How did the ancient scribes perform calculations of surfaces and volume using numbers without including orders of magnitude? The answer lies in school tablets unearthed at the sites of scribal schools from the Old Babylonian period (beginning of second millennium B. C.). I will show several instances of these pedagogical notes. I will also present some examples of computations produced in the same period, in order to show how a positional notation without zero is both a powerful calculation device and a source of errors. Finally, I will show how the zero was introduced during the Seleucid period (last centuries B. C.) in large numerical tablets coming from the Uruk and Babylon Libraries.

On s'intéressera dans cet atelier aux plus anciennes traces de zéros attestées dans les écrits mathématiques, et notamment à celles qu'on trouve dans la documentation mésopotamienne.
Quand et où voit-on apparaître un signe spécial pour indiquer une place vide dans la numération positionnelle? Pourquoi les Babyloniens n'ont-ils pas inventé le zéro en position finale? Dans quel contexte apparaît le zéro en tant que nombre? Les réponses à ces questions conduiront à éclairer les multiples facettes du zéro (chiffre en position médiane, chiffre en position finale, nombre), la diversité des problèmes de calcul auxquels répondent ces innovations indistinctement baptisées "invention du zéro", la très longue durée dans laquelle s'inscrit le processus d'émergence de la notion actuelle de zéro. On insistera particulièrement sur le rôle des algorithmes arithmétiques, des techniques de calcul, de l'usage d'instruments matériels (abaques, surfaces à calcul), dans l'évolution de l'écriture des chiffres et dans l'invention des différentes formes de zéros.
L'atelier s'appuiera sur des textes de Mésopotamie (tablettes scolaires d'époque paléo-babylonienne; tables numériques d'époque séleucide), d'Inde et de Chine anciennes.
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***Theme* 4**

HISTOIRES DE MATHÉMATICIENNES / WOMEN MATHEMATICIANS' HISTORY

Marie-Noelle Racine
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L'histoire n'accorde aux femmes qu'une place minime, même quand elles ont joué un rôle de premier plan, plus particulièrement dans le domaine des mathématiques. Cet atelier a pour but de faire connaître quelques noms, de replacer ces femmes dans leur contexte social, politique, mathématique, de faire travailler sur leurs écrits ou sur des mathématiques qu'elles ont pu pratiquer.
History grants only a small place to women, even if they played an important role, and more particulary in the field of mathematics. This workshop aims at making known some names, to replace these women in their social, political, cultural and mathematical context and to make work on their papers or on maths they may have practiced.

Back to 3-hour Workshops abstracts

Theme 5

***Theme* 5**

VARIATIONS IN MATHEMATICAL KNOWLEDGE OCCURRING IN THE MODERN MATHEMATICS REFORM MOVEMENT

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Under the denomination of Naturalistic Mathematics, David Bloor (1991) proposes an account of the nature of mathematical knowledge that incorporates contributions from J. Stuart Mill and Gotlob Frege. Mill proposes that mathematical knowledge comes from experience and Bloor argues that, as Frege points out, experience alone does not provide an adequate background for mathematical knowledge. “The characteristic patterns” of objects, as Mill puts it, are not on the objects themselves. These patterns are social, rather than individual, entities and they are at the very root of the objective objects of Reason proposed by Frege. Mill’s theory does not do justice to the objectivity of mathematical knowledge, to the obligatory nature of its steps, or to the necessity of its conclusions. This missing component is made of social norms that single out specific patterns, endowing them with the kind of objectivity that comes from social acceptance. Bloor proposes that “the psychological component provide[s] the content of mathematical ideas, the sociological component deal[s] with the selection of the physical models and accounted for their aura of authority” (p. 105). By extending Mill’s theory sociologically and by interpreting sociologically Frege’s notion of objectivity Bloor opens the door to what he calls “alternative mathematics”. Alternative mathematics would look as error to our mathematics. These errors should be “systematic, stubborn or basic” (p. 108) and they should be “engrained in the life of a culture” (p. 109). Bloor presents four types of variations in mathematical thought that can be related to social causes.
Commonly, however, mathematical laws are understood as absolute, and eternally true, learning mathematics is to understand something previously given with a clear distinction between right and wrong or true and false, distinct cultures contribute to the same pool of mathematical knowledge. These conceptions about the nature of mathematical knowledge exclude the possibility of variations.

This workshop will discuss the adequacy of the notion of variation in mathematical knowledge as a means to understand specific polemics among mathematics educators during the Modern Mathematics reform. Two distinct cases in Portugal will be studied. The session will have the following programme:
I. Theoretical background
a) a brief presentation of Bloor's ideas (20m)
b) an analysis of one of Bloor's cases (10m).
II. Modern mathematics in Portugal (10m)
III. Discussion between Cardoso and Gil
a) Analysis of the discussion (20m)
b) Debate (20m)
IV. Discussion between Nabais and Lopes
a) Analysis of the discussion (20m)
b) Debate (20m)
V. Final debate (60m)

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***Theme* 5**

THE "ELÉMENTS DE GÉOMÉTRIE" OF A. M. LEGENDRE: AN ANALYSIS OF SOME PROOFS FROM YESTERDAY'S AND TODAY'S POINT OF VIEW

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The "Eléments de Géométrie" of A. M. Legendre was largely adopted in many countries since his first edition in 1794. When around 1870 some countries, like Italy or England, decided to adopt Euclid's Elements much criticism raised against Legendre's book. The critics concerned mainly the use of algebraic means, and a general lack of rigor. Instead, no mention was made to Legendre's attempts to prove Euclid's 5th Postulate.
In the workshop we will analyse the use of arithmetic and algebraic means on the part of Legendre, also having in mind today's use of those means. We will also analyse Legendre's use of infinity when dealing with the 5th Postulate. We will also confront some proofs given by Legendre with those contained in the later Blanchet's edition, and discuss their didactical meaning in comparison to today's requirements.
The original text of Legendre is in French, we will try to provide the American edition too. The discussion will be mainly conducted in English.

Back to 3-hour Workshops abstracts

Theme 6

***Theme* 6**

FROM VITELLONIS' GEOMETRY TO UNRAVELLING THE SECRET OF ENIGMA – ON MILLENNIUM OF THE POLISH MATHEMATICAL THOUGHT

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Recently we are witnessing an extremly rapid civilization progress, mainly involved by dramatic development of information and communication technologies. New digital media change the face of today's science, technology, economic and social life. A possibility to use IT causes applying formal mathematical structures in all these areas, making their functioning more and more effective. In the knowledge-based society, understanding mathematics, using its language and methods must become not only an indispensable component of professional equipment but also a necessary condition of effective functioning in every day life. Thus, in a frame of general education „for all“ there is a strong need of developing mathematical thinking rather than mastering some routine skills.

In the proposed workshop I would like to invite participants to discuss a role of the knowledge on history of mathematics in a process of learning and teaching mathematics.

The most interesting issues which arise in relevance to this main aim of the workshop are the following:

How the knowledge on history of mathematics can influence people's attitude to mathematics education – from the point of view of students, teachers, parents, educators, public opinion;

What are stereotypes concerning mathematics education, which are connected with the knowledge on history of mathematics, and what is their impact on today's classroom practice;

How the knowledge on history of mathematics can improve the process of mathematics teaching according to students‘ cogntive development.

Participants of this workshop are invited to discuss these issues on the basis of some historical and didactical materials which are mainly taken from the Polish mathematics textbooks, addressed to pupils and students age of 10-19, and from publications addressed to teachers. During this discussion, on the ground of short information on historical development of Polish mathematical thought, participants will have an opprotunity to recognise various ways of using history of mathematics in order to make the process of mathematics learning more effective. Although the points of departure choosen for discussion base on the Polish experience, they will create a great opportunity to consider the proposed issues, evoking experiences of various countries. The mathematical content of examples presented to participants is related to the most fundamental mathematical concepts and competencies, such as: real numbers, probability, incommensurability, ability to prove, probabilistic and statististical thinking. All materials prepared for particpants to discuss will be translated into English.

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Back to 3-hour Workshops abstracts

Back to the Main Themes

2-hours Workshops (based on didactical and pedagogical material)

(Ordered alphabetically)

Name	Title	Theme	Language	Country
Ballieu Michel, Guissard Marie-France	Pour une culture mathématique accessible à tous	2	English & French?	Belgium
Caianiello Eva	Le problème d'oiseaux: procédés de résolution dans l'histoire des mathématiques	2	French	Italy
Dematté Adriano	Historical documents in everyday classroom work	2	English	Italy
Dias Isabel	From the original texts of Pedro Nunes to the mathematics classroom activities	2	English	Portugal
Dlab Vlastimil	Knowing, Teaching and Learning of Algebra	1	English	Canada
FitzSimons Gail	Mathematics and the Personal Cultures of Students	4	English	Australia

Hejny Milan, Stehlikova Nada	Didactic simulation of historical discoveries in mathematics	3	English	Czech Republic
Katz Victor	Historical Modules for the Teaching and Learning of Mathematics	2	English	USA
Kourkoulos Michael & Tzanakis Constantinos	A didactical approach to the introduction of Statistics, inspired by epistemological and historical considerations	2	English/ French	Greece
Maschietto Michela, Martignone Francesca	Activités avec les *machines mathématiques*	2	French	Italy
Movshovitz-Hadar Nitsa	Incorporating mathematical news in teaching high school math and in teacher preparation	3	English	Israel
Paschos Theodorus, Farmaki Vasiliki	The integration of genetic moments in the History of Mathematics and Physics in the designing of didactic activities to introduce first-year University students to concepts of Calculus	2	English	Greece
Roelens Michel	Le volume d'une pyramide à travers les siècles: des tranches ou pas de tranches, voilà la question!	1	French/English	Belgium
Rogers Leo	Early Methods for Solving Real Problems	3	English	UK
Smestad Bjørn	Various material for primary school teacher training	3	English	Norway
Smid H.J.	Heuristic math education in the 19th century: a dead end or preparing the ground?	5	English	Netherlands
van Maanen Jan	The work of Euler and the current discussion about skills	6	English	Netherlands
Weeks Chris	Condorcet's paradox: a little history and some school activities	2	English	UK
Wilson Robin	Solving Dotty Problems	2	English	UK
Winicki Landman Greicy	Playing with fractions a la Leibnitz	2	English	USA
Yamalidou Maria	Mathematics as a way of "reading" the world: Thoughts about the underpinnings of interdisciplinary teaching	1	English/ French	Greece

Back to the Main Themes

2-hours Workshops ABSTRACTS

(ordered by theme, 1, 2, 3, 4, 5, 6)

Theme 1

***Theme* 1&3**

KNOWING, TEACHING AND LEARNING ALGEBRA

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The main objective of this Workshop is to bring to the schools (at all levels) the basic rules of learning and teaching – imitating the process by which children naturally learn new concepts. This requires a thoughtful build-up of concrete carefully chosen examples enriched by comments to relevant historical developments, well before embarking on any generalizations (often graciously called abstraction).
The Workshop will provide for all participants a booklet of carefully chosen examples and problems covering all levels of pre-university schooling. They will be chosen to reflect the above principles of teaching, to broaden the supply of class material, and as such will be thoroughly discussed. They will even contain several questions from some earlier educational meetings that were not sufficiently illuminated. The examples will blend algebra with geometry and number theory, thus emphasizing the Unity of Mathematics.
Excellent teaching requires excellent (and enthusiastic) teachers, armed with a broad knowledge of basic Mathematics. Only they can make learning Algebra, and indeed Mathematics, attractive. It should be one of

theirs main goal to revive the Algebra textbooks that have been presently degraded to lifeless collection of definitions and trivialities. Therefore the second part of the Workshop will be dedicated to questions related to education of the future teachers of Mathematics. It will try to assist them in preparing for their profession and provide some hints for upgrading of teaching of Algebra. Teaching of Mathematics should never be degraded to providing instructions!
Trying to assist in attracting students to Mathematics, the Workshop will also attempt to answer the following question: What do we do so poorly in our schools – that we drive children away from Mathematics?
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***Theme* 1**

LE VOLUME D'UNE PYRAMIDE A TRAVERS LES SIECLES: DES TRANCHES OU PAS DE TRANCHES, VOILA LA QUESTION!

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Bien avant l'âge d'étudier le calcul intégral, les élèves apprennent à utiliser les « formules » permettant de calculer le volume d'une pyramide, d'un cône, d'une sphère, … Le volume de la pyramide ou du cône est égal à un tiers de l'aire de la base multipliée par la hauteur; le volume de la sphère est égal à un tiers de son aire multipliée par le rayon. Mais pourquoi un tiers? On convainc les élèves en versant de l'eau: le contenu de trois cônes creux égaux remplit exactement un cylindre de même base et hauteur. Exactement, ou presque, à une goutte près? De plus, ceci ne répond pas vraiment à la question « pourquoi » un tiers. Alors, on leur présente les solides comme composés de feuilles de papier ultrafines, et on fait recours à un raisonnement dit « de Cavalieri », où l'on saute du deux-dimensionnel au trois-dimensionnel. «Le volume d'une feuille ultrafine, est-ce son aire ? » C'est tout de même bien plus compliqué que l'aire du triangle, où on y arrive par assemblage et découpage sans sauter les dimensions.
Dans cet exposé-atelier, nous parcourrons l'histoire du volume des solides élémentaires et de l'explication du facteur «un tiers». Nous verrons qu'un volume n'a pas toujours été considéré comme un «nombre que l'on calcule à l'aide d'une formule». De plus, nous discernerons deux tendances opposées dans cette histoire : d'une part le recours aux tranches ultrafines à la manière de Cavalieri, et d'autre part les efforts pour éviter ce recours. Comme l'a démontré Max Dehn en réponse au troisième problème de David Hilbert, ces tentatives n'aboutiront jamais vraiment à une solution par assemblage et découpage comme dans le cas du triangle plan.
Cet atelier est destiné à des (futurs) professeurs d'élèves de 12 à 16 ans.

The volume of a pyramid through the ages: slices or no slices, this is the question!

Long before the age of studying integral calculus, the pupils are taught to use "formulae" in order to calculate the volume of a pyramid, a cone, a sphere... The volume of a pyramid or a cone equals a third of the area of the basis multiplied by the height; the volume of a sphere equals a third of its area multiplied by the radius. But why a third? The teacher convinces the pupils by pouring water: the content of three equal hollow cones fill exactly a cylinder with the same basis and height. Exactly, or almost exactly, except for a droplet? Moreover, this really does not answer the question "why" a third. Therefore, teachers or text books present the solids as made up of very thin sheets, and produce a reasoning known as "of Cavalieri", jumping from the two-dimensional to the three-dimensional. "The volume of a very thin sheet of paper, is it its area?" It seems to be much more complicated than the area of a triangle, where one gets there by assembling and cutting without jumping dimensions.
In this workshop, we will go across the history of the volume of the elementary solids and the historical explanations of the factor "one third". We will see that a volume has not always been seen as a "number that one calculates using a formula". Moreover, we will distinguish two opposite tendencies in this history: on the one hand the recourse to the very thin slices in the Cavalieri style and on the other hand the efforts to avoid this recourse. As it has been demonstrated by Max Dehn in response to David Hilbert's third problem, these attempts will never really lead to a solution by assembling and cutting as in the case of the plane triangle.
The workshop is intended for (future) teachers of 12-16 years old pupils.
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***Theme* 1**

MATHEMATICS AS A WAY OF "READING" THE WORLD: THOUGHTS ABOUT THE UNDERPINNINGS OF INTERDISCIPLINARY TEACHING

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The aim of this workshop is to clarify the characteristics and basic presuppositions of two distinctive approaches to interdisciplinary teaching, which I call the "summative" and the "superimpositional" approach respectively. The summative approach seems to be the impulsive response of Grammar School teachers in Greece, as they attempt to come to terms with the newly introduced, diathematic curricula. According to this approach, a topic is chosen and then the teacher tries to induct in the discussion as many of the taught disciplines as possible. The use of mathematics in this approach is purely instrumental and has to do merely with the computation of any quantitative parameters that may arise in the chosen topic. The role of the other disciplines is equally limited, as this approach seems to focus merely on the stitching together of different pieces of previously acquired knowledge.

I wish to argue that, in order to gain full advantage of interdisciplinarity, teachers need to try to materialize the known dictum that the whole is more than the summation of its parts. At the epistemological level, this means a total transformation of the teachers' understanding of the scientific and non-scientific disciplines they teach, not as pieces of knowledge, whose accumulation provides a true picture of the world, but as different systems of knowledge, which suggest different, complementary interpretations of the world. This understanding can be best materialized by the "superimpositional" approach. In this approach the different disciplines *qua* systems-of-knowledge are superimposed on each other, like different transparencies which allow the (mind's) eye to get and hold the images of all and each. The main advantage of such teaching is this: that it retains the complexity of the world and helps pupils decide which are, each time, the most appropriate resources for understanding this complexity. According to this approach, mathematics ceases to be merely a tool for measuring the world; it becomes a distinct way of reading the world, which enhances pupil's understanding of the other systems of knowledge. Historically, the mathematization of the world has been a process that aimed at its conceptual understanding and it is sad that we have not yet found the way to appropriate this historical reality in the classroom.

The superimpositional approach to interdisciplinary teaching seems to provide a framework for a better integration of mathematical knowledge within the cognitive space of pupils. But in order for this approach to be productive, we need to answer questions such as: what is it that would create the need to move form a poetical, for example, to a mathematical understanding of the world? Through what kind of questions can we induce the idea that pupils can understand the world better if they mathematize it? This line of thought will force us to explore the questioning activity as a cognitive process that allow pupils to construct their own universe of meanings about the world, thus becoming active thinkers.

Back to 2-hour Workshops abstracts

Theme 2

Theme 2

POUR UNE CULTURE MATHEMATIQUE ACCESSIBLE A TOUS

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Resorting to cultural activities can prove to be invaluable to introduce and install abstract notions. This workshop emphasizes two means for restoring the pleasure of learning to demotivated pupils: history and artistic realizations.

The historical approach of mathematics allows to enter the concepts showing in which context and why they were born, how they have evolved. A round through the systems of numeration and the solution of equations gives a good example of these words. As to geometrical decorations whose examples are founded into all civilizations they can be used as aid for geometry learning; so geometry shows its whole visual attraction. Repetitive patterns as friezes or tilings lend themselves to activities, which combine intuition, creativity and analysis of mathematical structures.

Le recours à des activités culturelles peut s'avérer une aide précieuse pour introduire et installer des notions abstraites. Cet atelier met l'accent sur deux registres susceptibles de rendre un certain plaisir d'apprendre aux élèves démotivés: l'histoire et les réalisations artistiques.

L'approche historique des mathématiques permet d'aborder les concepts en montrant dans quel contexte et pourquoi ils sont nés, comment ils ont évolué. Un parcours à travers les systèmes de numération et la résolution des équations illustre ce propos. Quant aux décors géométriques, dont on trouve des exemples dans toutes les civilisations, ils peuvent servir de support à l'apprentissage de la géométrie, qui montre ainsi tout son attrait visuel. Des motifs répétitifs tels que les frises ou les pavages se prêtent à des activités qui allient intuition, créativité et analyse des structures mathématiques.

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Theme 2

LE PROBLEME D'OISEAUX : PROCEDES DE RESOLUTION DANS L'HISTOIRE DES MATHEMATIQUES

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Sous la dénomination de «problèmes d'oiseaux ou de volatiles»,on se réfère à l'achat, avec une somme connue, de différents types de volatiles dont on connaît le nombre total et le prix à la pièce. Les problèmes d'oiseaux

appartenaient à un type de problèmes indetérminés, d'origine très ancienne, que l'on rencontre d'abord en Chine, dans l'œuvre de Zhang Qiujian (milieu du 5e siècle de notre ère), en Inde dans le manuscrit de Bakhshili (7e siècle?) et dans le Compendium de l'essence de la mathématique de Mah?vir? (milieu du 8e siècle), en Egypte dans le livre des choses rares de l'art du calcul d'Ab? K?m?l vers 900 et en Europe dans les *Propositiones ad acuendos juvenes* d'Alcuin de York en 800 environ. Dans le monde musulman il faisait partie des problèmes de la Mu'?mal?t arabe, i.e. la science du calcul appliquée à l'art du commerce et de problèmes de transaction.. Comme jeu récréatif, le problème était très diffusé à toutes les époques ce qui expliquerait l'intérêt que lui portent des personnages de la cour de l'empereur Frédéric II de Hohenstaufen (1194-1250), comme le maître Théodore d'Antioche, philosophe de l'empereur, qui fut destinataire d'un opuscule composé par Léonard de Pise (dit Fibonacci) après 1228, la Lettre a maître Théodore où le mathématicien traite ce problème. La procédure employée par Léonard de Pise dérive des règles d'alliage. Nous traiterons ici :
1. De l'histoire du problème et de ses procédés de résolution à partir de la formulation de Zhang Qiujian ;
2. De l'usage de la 6ième règle de mixage par Léonard de Pise.3. En conclusion, nous soulignerons de ressemblances entre les procèdes indiens de Mah?vir? et ceux de Léonard de Pise.
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***Theme* 2**

HISTORICAL DOCUMENTS IN EVERYDAY CLASSROOM WORK

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A book (edited by me) is a collection of passages taken from original sources. The title is *Fare matematica con i documenti storici* (Doing mathematics with historical documents). "Fare matematica" ("doing mathematics") in the title stresses the fact that this book is not only for "reading on mathematics" but rather for operating with exercises and problems. The aim of this book is furnishing secondary school teachers with a proposal of activities to integrate originals in everyday classroom work. This integration should promote alternative ways of teaching based on working with texts and exercises to reinforce (or sometimes even to introduce) mathematical competencies.
The publication is the product of two years work of five in-service teachers. They collaborated in a facultative way with IPRASE Istituto Provinciale di Ricerca, Aggiornamento e Sperimentazione Educativi del Trentino (Provincial Educational Research Institute, Trento – Italy). IPRASE is not specifically mathematics-addressed and its aim is to improve school quality in a 400 000-inhabitants alpine province. The history of mathematics, and the use of original sources in the classroom specifically, were considered in their educational potentiality.
Documents have been chosen in order both to offer many important authors and to show less famous mathematicians whose works were representative in their living time. Main Italian secondary school topics are present. Questions and activities follow every document: their role is addressing the student's attention toward the main points in the original document, involving her/him in an active way. Some question requires further exercises and problems, using different examples or recalling themes in mathematics textbook. Sometime students are asked to conjecture about the causes of certain historical facts. Although we know they don't have a real expertise, students could consider what might have been the antecedents.
Volume *b* is specifically teacher-addressed. It contains didactical suggestions, solutions to exercises, some further going themes and a bibliography.
Material that is going to be used for the workshop: selected worksheets for students (English version) from the book *Fare matematica con i documenti storici*; selected pages (English version) from the volume for teachers; questionnaires for participants.
The workshop refers to aged 12-18 students.

[1]Dematté, A. (editor), a, 2006, Fare matematica con i documenti storici – una raccolta per la scuola secondaria di primo e secondo grado. Volume per gli studenti, Editore Provincia Autonoma di Trento – IPRASE del Trentino.
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***Theme* 2**

FROM THE ORIGINAL TEXTS OF PEDRO NUNES TO THE MATHEMATICS CLASSROOM ACTIVITIES

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"The discoveries are a phenomenon of worldwide European expansion during the fifteenth and sixteenth centuries, in which Portugal played a fundamental and pioneering role" (Barreto & Garcia, 1994, p.18). The astronomical and mathematical problems related to the navigations were part of the seamen daily life. But those questions, which were put in a simple way by the Portuguese sailors, gave rise to a new field in science. Men like Duarte Pacheco Pereira (1460-1533), D. João de Castro (1500-1548) and, above all, Pedro Nunes (1502-1578), discussed "the declination of the nautical compass, cartographic projection, the creation and perfection of

instruments for measuring height and tables of latitudes, the theory of tides and theory of the proportional division of the globe between land and sea" (Barreto & Garcia, 1994, p.52).
The outstanding role of Nunes in the mathematics of the 16th century has been recognised by Portuguese and international researchers (Albuquerque, 1988; Hoyrup, 2002; Katz, 1998; Stockler, 1819). Actually, he was the author of *Tratado da Sphera*, *De Crepusculis* and *Libro de Algebra en Arithmetica y Geometria*, among other books that deeply shaped the scientific thought of his time.
In fact, the great maritime voyages of the Portuguese would not have been possible without major technical developments in the art of building the ships, in the cartography and in the nautical and astronomical devices. Instruments like quadrants and nautical astrolabes provided information about the height of the stars and its accuracy depended on the scale precision. Nunes theoretically genial idea to improve the precision of a quadrant gave rise, through Christoph Clavius and Pierre Vernier, to the rectilinear instrument that allows measuring the smallest objects with great accuracy, "Nonius". Another instrument was imagined by Pedro Nunes: the "instrument of shades" which, although a quite simple device, it is a rather tricky one: a triangle shadow is simply transferred to a graduated circle upon a horizontal base. D. João de Castro, the Portuguese nobleman that was the commander of one of the Portuguese fleets that reached India, experimented the instrument. His notes about the results of the tests proved its amazing precision. The "nautical ring" was another of the instruments imagined by the Portuguese astronomer. In his book *De arte atque ratione navigandi libri duo* he described a ring that, in spite of being an interesting application of simple geometrical facts proved by Euclides, it wasn't really accurate.
The proposed workshop intends to outline the role of Nunes in the 16th century mathematics and to analyse some geometrical aspects of his work (through the visualisation of a small Power Point show) but, essentially, its main aim is to give the workshop attendance the opportunity to experiment the practical activities and to discuss their use in classroom. The pedagogical material used in the workshop refers to students between 12 and 16 years old and each worksheet has classroom notes and suggestions to teachers. We hope we can also discuss the epistemological and didactical consequences of using this kind of historical material in mathematics classrooms.

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***Theme* 2**

HISTORICAL MODULES FOR THE TEACHING AND LEARNING OF MATHEMATICS

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The CD entitled *Historical Modules for the Teaching and Learning of Mathematics* was developed to demonstrate to secondary teachers how to use material from the history of mathematics in teaching numerous topics from the secondary curriculum. Developed by secondary and college teachers working together, this CD contains eleven modules dealing with historical ideas directly usable in the secondary classroom. The modules are in Trigonometry; Exponentials and Logarithms; Functions; Geometric Proof; Lengths, Areas, and Volumes; Negative Numbers; Combinatorics; Statistics; Linear Equations; Polynomials; and a special module on the work of Archimedes. Each module contains numerous activities designed to be used in class with minimal further preparation from the teachers. A given activity contains instructions to the teacher as well as pages for distribution to the students. The teacher instructions discuss the rationale for the activity, its placement in a class, the necessary time frame (which may be as short as fifteen minutes or as long as two weeks), and the materials needed. They also contain historical background, masters for making transparencies, and, if necessary, answers to student exercises. The student pages may discuss the historical background of the particular topic, lead the students through the historical development, provide exercises and additional enrichment activities, and provide pictures and biographical sketches of mathematicians. They also provide references for further study, including both print and electronic material.
In the proposed workshop, the project director will discuss the CD with its wealth of materials and lead the participants through selected activities. These activities will include some that can be used at the beginning of secondary school, such as material on measurement in ancient societies, some that are appropriate for standard

secondary courses, such as ideas on solving quadratic and cubic equations, and some that are suitable for advanced secondary or beginning university students, such as the development of the power series for the exponential function. The director will also lead a discussion on the rationale for using historical materials in class as well as on the varied ways teachers can use the materials on the CD. In addition, he will discuss some results based on work with material in these modules with teachers and students in various settings. Each workshop participant will receive a copy of the CD for use in his/her own classes.
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***Theme* 2**

A DIDACTICAL APPROACH TO THE INTRODUCTION OF STATISTICS, INSPIRED BY EPISTEMOLOGICAL AND HISTORICAL CONSIDERATIONS

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Historical investigations indicate the existence of an intimate relation between Statistics and Physics during their historical development. Notably, throughout the 18th and at the beginning of the 19th century, the development of methods concerning the treatment of variation was mainly realised in the context of solving problems in Geodesy and Astronomy, where measurements necessarily involved errors. The relevant physical framework of Newtonian mechanics played an important role in this development: it provided physical meaning to the emerging methods, orienting and/ or justifying their formation, as well as the formation of the related statistical concepts). Subsequent developments, which characterized the historical development of both Statistics and Physics during the 19th and until the beginning of the 20th century, have gradually permitted us to understand that basic statistical concepts (such as the sum of squares distances from a center or from a regression line, the binomial and the normal distribution) have a deep physical meaning and were involved in the modelisation (and thus in the understanding) of fundamental Physical phenomena (such as the absolute temperature of ideal gases and solid bodies, the Brownian motion, etc).
In the usual, introductory teaching of statistics, very few -if any- references to physical models are being given. This means that the didactical implications of the historical relations between Statistics and Physics are not being taken advantage of. This lack of discussion of physical models appears even more important in the light of didactical studies which indicate that when students are introduced to statistics, they encounter substantial difficulties in understanding basic statistical concepts and methods (e.g. the measures of central tendency and of dispersion or the method of least squares).
In our presentation we shall trace possible didactical opportunities offered by the use of physical models while teaching introductory courses of Statistics both to high school students and university undergraduates and we shall highlight the fact that this combined approach can significantly enhance their understanding of some basic statistical concepts (e.g., the variance, the method of least squares). To the participants of the workshop we shall make available material concerning both the a priori didactical and epistemological analysis and the three years long, experimental teaching to prospective primary school teachers.
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***Theme* 2**

ACTIVITES AVEC LES *MACHINES MATHEMATIQUES*

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Une *machine mathématique* est un artefact réalisé avec un but précis, qui ne dépend pas de son usage effectif (s'il y en a un) : il oblige un point, un segment ou une figure plane à être transformé en accord avec une loi mathématique définie par le constructeur. Une machine mathématique peut être un traceur de courbe, un pantographe pour les transformations géométriques ou un outil pour dessiner en perspective. Elles ont été construites (pendant vingt ans d'activité par l'équipe de recherche en didactique des mathématiques de l'UFR de Mathématiques de l'Université de Modena-Reggio Emilia) avec un but didactique, en accord avec les descriptions contenues dans des textes historiques à partir des textes grecques (les traités des coniques) jusqu'au XXème siècle. Diverses machines sont présentes dans le Laboratoire des Machines Mathématiques (http://www.mmlab.unimore.it), qui est situé dans les locaux de l'UFR de Mathématiques de l'Université de Modena-Reggio Emilia. Les machines mathématiques sont considérées dans des projets de recherche en didactique des mathématiques visant à étudier l'enseignement et l'apprentissage de la géométrie à travers l'interaction avec certains artefacts. Les expérimentations conduites dans les classes ainsi que l'activité d'accueil de classes dans le Laboratoire des Machines Mathématiques même ont permis de mettre au point certaines activités et certains parcours pour les élèves. Le but de l'atelier est celui de les faire connaître et de explorer certaines machines mathématiques.
L'atelier est structuré en trois parties.
Dans la première partie, nous présentons les activités organisées dans le Laboratoire pour les classes de l'enseignement secondaire. On propose de séances de travail avec les machines mathématiques sur deux thèmes

(un troisième sera proposé à partir de la rentrée 2006) : sections coniques et transformations géométriques. Chaque séance est organisée en trois moments : introduction au thème de la séance, travail des élèves en groupe sur des machines mathématiques concernant le thème, présentation du travail en groupe.
Dans la deuxième partie, des activités sur les machines mathématiques sont proposées aux participants. Cette partie termine avec la présentation des machines étudiées. Les instruments présents dans l'atelier concernent les sections coniques et les transformations géométriques.
La troisième partie est représentée par un moment de discussion sur le travail proposé.
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***Theme* 2**

THE INTEGRATION OF GENETIC MOMENTS IN THE HISTORY OF MATHEMATICS AND PHYSICS IN THE DESIGNING OF DIDACTIC ACTIVITIES TO INTRODUCE FIRST-YEAR UNIVERSITY STUDENTS TO CONCEPTS OF CALCULUS

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History of Mathematics can play an important role in Mathematics Education. The debate about the benefits and difficulties, that can possibly result from the integration of history in the educational practice, becomes continuously more intense and essential.
We designed a series of activities aiming to introduce first-year undergraduates to basic Calculus concepts (the integral and Fundamental Theorem of Calculus), inspired from History [14th century (Merton College, N. Oresme), 17th century (G. Gallilei), method of exhaustion (Eudoxos, Archemides)]. We adapted the idea of integrating the history of Mathematics into the educational praxis, mainly utilizing 'genetic' ideas in the study of motion - a study between mathematics and physics during the later middle ages - which constituted crucial steps in the construction of these concepts.
We used a teaching approach, which has been described as a *genetic approach* to teaching and learning. In our research, following the general sense of this approach: (1) we identified the genetic historical 'moments', emerging during the 14th century in embryonic form, of some basic mathematical concepts, like a function of continuous variation and its graphical representation, the derivative, the definite integral and their relation represented geometrically and kinetically; and, (2) we reconstructed these crucial ideas in a modern version, suitable for classroom use.
The educational designing of the activities was based on motion problems and mainly on the velocity – time representation on Cartesian axes, in which velocity, time, and distance covered appear simultaneously, with distance represented as the area of the figure between the curve and the time axis. By interrelating the distance covered with the areas of the corresponding figures, the students are led to realize the connection between velocity and distance covered in the same graph, and thus to grasp the essential point of the fundamental theorem of Calculus.
The educational intervention was a part of a wider action research aiming to study the difficulties students faced, bridging the gap between intuitive, informal and formal mathematical knowledge. The activities were presented to first-year undergraduates of the Mathematics Department of the University of Athens, during the summer semesters of 2001-2002 and 2002 -2003 as an introduction to Integral Calculus. The instructive approach, which consisted of ten 'one-hour' teaching sessions, was applied in an interactive milieu where the students worked in pairs in the classroom, using work-sheets.
In order to investigate students' difficulties, their mental operations and understanding, some of them were interviewed individually.
In the work shop we will present: (1) some elements of the History of Mathematics and Physics concerning the study of motion on which we based the designing of the activities [14th century (Merton College, N. Oresme), 17th century (G. Gallilei)], (2) the work –sheets of the activities presented to the students, (3) some excerpts of the students' interviews, and (4) some observations of the analysis of the data collected.
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***Theme* 2**

CONDORCET'S PARADOX: A LITTLE HISTORY AND SOME SCHOOL ACTIVITIES

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Following the Revolution, Condorcet was a key player in the creation of a new social system for France. He was also innovative in developing an interest in applying mathematics to social questions, his *Essai* on probabilities of voting systems raising important questions about decidability. Here he demonstrates a contradiction that can arise in a simple voting system, which has come to be known as Condorcet's paradox. In probability theory this means there can be systems where A > B, B >C, C > A can all be simultaneously true.

In the workshop there will be an opportunity to read parts of Condorcet's *Essai* (with English translation and commentary). The underlying simple probability theory will be explained and examples of contradictory systems will be generated.
The purpose of the workshop will be for the participants to generate activities suitable for their own classroom, including elementary probability. There are obvious cross-curricular opportunities e.g. French language, history, current affairs.
Finally, I will report on using the material in English classrooms.
Level: secondary school, teacher training
References:

Condorcet, *Essai sur l'Application de l'Analyse à la Probilité de Décisions Rendue à la Pluralité des voix*, Paris, 1785.
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***Theme* 2**

SOLVING DOTTY PROBLEMS

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In this 2-hour workshop I introduce some of the ideas of graph theory through recreational puzzles, setting each in its historical context. Topics covered include traversability (the Konigsberg bridges problem and the icosian game), trees (chemical molecules), planarity (the gas, water and electricity problem) and colouring (the map-colouring problem).
[This is suitable for all ages – for students (aged 12-21) and for teachers. I shall introduce the puzzles historically, then give out some worksheets with related puzzles, and finally go through some methods for solution.]

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***Theme* 2**

PLAYING WITH FRACTIONS A LA LEIBNITZ

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In this workshop, elementary and middle school teachers are going to play with an array of numbers considered by the German mathematician G. W. Leibnitz in the beginning of the XVIII century. This array of rational numbers is mathematically very rich and its investigation will be the main topic of the workshop. This richness consists of multiple possibilities of looking for patterns, formulation of conjectures, searching for analogies and making connections. A genuine mathematical investigation that may introduce young students to the need of using variables to describe mathematical patterns and to the different roles played by proofs in the mathematical endeavour.
The idea of series will also be discussed and the purpose of Leibnitz's work on this array will also be analyzed.
This workshop illustrates a concrete way of adding an historical dimension to the teaching of mathematics, especially when looking for significant tasks for young learners.

Back to 2-hour Workshops abstracts

Theme 3

***Theme* 3**

DIDACTIC SIMULATION OF HISTORICAL DISCOVERIES IN MATHEMATICS

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The workshop is aimed for students – future mathematics teachers and future elementary teachers and also for practicing teachers teaching talented secondary school students. We will prepare worksheets with problems to solve, however, the participants might also be asked to pose their own problems.
Background: The history of mathematics is mostly considered to have a motivational power. Sometimes it inspires the authors of textbooks and curricula for using historical notes in teaching. Rarely, however, is the history of mathematics used in teaching directly. One of the ways is the way of simulation, or more precisely, the way of projection of phylogenesis to onthogenesis by simulation.
The main idea of the workshop is to create such conditions for students, which led to deep mathematical ideas in the history of mathematics. This is done by introducing them to a non-traditional context and giving them a series of carefully selected graded problems which should lead to

a) acquiring basic experience with the context (students acquire isolated models of knowledge)
b) finding out some relationships (students acquire generic models)
c) finding out some structural ideas
d) creating their own problems which enable them a further insight into the structure
e) formulating definitions of concepts, hypotheses and their proofs.

Two contexts will be presented:

1. a non standard finite arithmetic structure in which by means of solving linear (and later on also quadratic) equations, a student step-by-step constructs particular concepts such as identity, inverse, group, subgroup, ring, ideal,…

2. a trileg mini geometry with a simple set of theorems which allows a solver to create his/her own axiomatic system, to solve the problem of independency of the set of axioms and to find a proof of unconstructability of some objects.

In both contexts, only the secondary school mathematics is used, there is no need to apply university mathematics. These contexts have been widely used in our research with future mathematics and elementary teachers.

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***Theme* 3**

INCORPORATING MATHEMATICAL NEWS IN TEACHING HIGH SCHOOL MATH AND IN TEACHER PREPARATION

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Mathematics is a highly prolific discipline, with a vivid present and an as yet unforeseen future. New findings are published on a regular basis in the professional journals. Yet unsolved problems awaiting the motivated mathematician to cope with. However school mathematics all over the world does not reflect the ever growing nature of the field, nor the steady struggle of mathematicians for solving open problems and establishing new results. Consequently, high school graduates leave school having the wrong image of math as a discipline in which all answers are known, and there is very little room for further exploration. This non-constructive conception of mathematics is henceforth spread around to the public and it keeps the majority hating it on the one hand while admiring those weird ones who find it intriguing, on the other.

In my workshop I'll present a sample of fascinating and accessible math news, e.g. The proof of Kepler's conjecture about sphere packing, published in November 2005 after 7 years of debate by the Annals of Mathematics editorial referees; The endless race for higher prime numbers, in particular The Great Mersenne Prime Search (GIMPS) that revealed in December 2005 a discovery of the 43rd Mersenne prime, an almost 10 million digit prime, but not quite, hence the prize for getting over this size is still waiting…; Random walks recent results by Yi-Sun, a high school student from California, which brought him the Intel Talent Search award for 2005.

Workshop participants will collaborate looking for updated math news on the web, discuss the need, values and the appropriate pedagogy for introducing math news in the classroom, and develop methods to incorporate them in teacher training and in the math class-room.

My goal in this workshop fits the ESU Aim and Focus statement to " lead to a better understanding of mathematics itself and to a deeper awareness of the fact that mathematics is not only a system of well-organized finalized and polished mental products, but also a human activity, in which the processes that lead to these products are equally important with the products themselves."

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***Theme* 3**

VARIOUS MATERIAL FOR PRIMARY SCHOOL TEACHER TRAINING

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In the study year 2006-2007, I will include history of mathematics in my pre-service course for primary school teachers. These will include topics from the history of numeral systems and history of algebra, as well as connections to art.

The worksheets etc will be created throughout the year, and is therefore not available at the time of submitting the abstract (May 15th, 2006).

Back to 2-hour Workshops abstracts

Theme 4

***Theme* 4**

MATHEMATICS AND THE PERSONAL CULTURES OF STUDENTS

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One important aspect of teaching mathematics is to stress the harmony of mathematics with other intellectual and cultural pursuits. The history of mathematics reflects its origins as a human activity as people sought to make sense of their world — from the earliest utilisation of primitive symbolic systems in order to overcome the limitations of human memory. In more recent times, with the spread of universal schooling, in more developed countries at least, formal education processes for children and adolescents have generally followed the artificial separation of disciplines which originated with the medieval universities.

It is a commonly recognised phenomenon today that school students — and vocational students in my experience — carry this arbitrary separation of disciplines into their thinking processes and are unable, even unwilling, to (re)make the connections that might be logically present. So, although they can successfully complete assigned tasks in the mathematics or statistics classroom, when they are confronted with textual or practical applications in their other studies or even outside of school they are unable to competently draw upon mathematical knowledges and skills to creatively solve real problems situated in these different contexts. It is also commonplace that, in English-speaking countries at least, many adults from all walks of life claim both (a) not to have been good at mathematics and (b) that they never use any mathematics they learned in school. These two aspects are very sad reflections on a near-universal education system that encourages, even enforces, separation of discipline areas.

Following Bernstein's (2000) analysis, the school subject of mathematics is strongly classified — that is, there are very strong boundaries around what is considered mathematics and what is not. This is in contrast to the social sciences, for example. Following Bernstein, I have also argued elsewhere (FitzSimons, 2005) that the vertical discourse of mathematics is strongly contrasted with the horizontal discourse of (adult) numeracy, and that pedagogy which is only concerned with the former will not guarantee numerate behaviour in practice.

Another unintended outcome of the arbitrary separation of disciplines is that people may fail to appreciate fully the aesthetics of the natural environment or their cultural environment (e.g., music and other performing arts, visual and literary arts, history, architecture, etc.), because they have never been encouraged to connect different ways of knowing or to reconcile different forms of meaning in mathematics classes.

In this workshop I propose to share and explore topics, appropriate to the different developmental levels of learners of all ages, which might encourage boundary crossing. These involve focusing on economic, social, cultural, natural, and historical themes. My concern is that, wherever possible, mathematics should be seen by students to be immediately relevant to their lives, and as supporting them to make decisions that affect them personally.

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Workshop Materials: I will briefly introduce some activities I have developed with my own vocational students as exemplers.I will then present outlines of suggested didactical activities for participants to: (a) suggest alternative themes of more relevance to their own learners & (b) work with others or alone to develop useful materials (e.g. detailed plans, worksheets). The session will conclude with volunteers sharing what they have achieved and reflecting on what they learned in this session.

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***Theme* 4**

EARLY METHODS FOR SOLVING REAL PROBLEMS

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Were you taught to find square roots by hand when you were at school?

Did you understand the procedure, and do you still remember how to do it?

Is it still useful today? This is just one example of 'old fashioned' arithmetic that has fallen out of use due to the advent of cheap calculators.

Our collection of school arithmetic methods originate from places in the Ancient Middle East, India and China. These were compiled by unknown authors into oral and manuscript form known as the 'abacus tradition' which were gradually brought into printed form from the time of Leonardo of Pisa.

The workshop will look at a selection of typical problems which gave rise to techniques in elementary arithmetic, geometry, and proto-algebra which can be found in manuscripts and books dating from the thirteenth to the eighteenth century.
Some examples are:
The Rule of three, Calculation of plane areas, Division into pre-determined unequal parts (inheritance problems), Ratios & proportions, Calculation of volumes, Double false position, Problems of excess & deficit, Barter and exchange, Systems of linear equations, Procedure of the base and height (Pythagoras), Extraction of square and cube roots, and Square roots by construction.
Participants are invited to bring (or remember!) their own 'old fashioned' methods for discussion and comparison, to consider how many of these methods still remain in our mathematics curriculum, and which may still be useful in our society today.
For Teachers and Teacher Trainers of Primary and Secondary Pupils
Examples of original problems and background and notes on solution methods will be provided.

Back to 2-hour Workshops abstracts

Theme 5

Theme 5

HEURISTIC MATH EDUCATION IN THE 19TH CENTURY: A DEAD END OR PREPARING THE GROUND?

Harm Jan Smid
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In the 19th century, discovery or *heuristic* learning was advocated by many pedagogues, as for example F.A.W. Diesterweg. The idea was one of the consequences of the didactic developments in the beginning of the 19th century, as embodied for instance in the person of Pestalozzi. These developments had their greatest influence in the field of primary education, but there were some influences on secondary education. In The Netherlands, the idea of heuristic mathematics teaching in secondary schools was strongly propagated by J. Versluys in his book *Methods for teaching mathematics and its scientific treatment* (1874) Two years before, Willem Versluys, Jans brother, had published a textbook on plane geometry for Dutch secondary schools, titled : *Concise textbook of plane geometry on a heuristic basis.* In his foreword he mentioned textbooks by Karl Snell and Oskar Schlömilch, who wrote textbooks on a similar basis. But by and large, during the 19th and a large part of the 20th century, heuristic math teaching in secondary education remained rather obscure. Traditional teaching prevailed.
In the last decades of the 20th we witness a revival of heuristic or discovery math learning and teaching. Is there any connection with the ideas from the 19th century?
In this workshop we will take a closer look to those 19th century math textbooks on a heuristic basis. After a short introduction we will read and discuss fragments of the textbooks of Snell, Schlömilch and Versluys (a translation will be provided). We will compare them with more traditional 19th textbooks and with modern heuristic textbooks.
Another aim of this workshop is to set up an inventory of 19th century math textbooks on a heuristic basis in other countries. Participants are therefore invited to inform the workshop of titles of textbooks of this kind they are aware of, or if possible, to bring (fragments of) this textbooks with them.
The workshop is intended for secondary math teachers, students of teacher training institutes and more generally for everybody interested in math teaching in secondary education.
No special knowledge, apart from knowledge of the traditional curriculum on geometry, arithmetic and algebra for secondary schools, is required. Some texts will be in German.

Theme 6

Theme 6

THE WORK OF EULER AND THE CURRENT DISCUSSION ABOUT SKILLS

Jan van Maanen
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This session will be a combined workshop and lecture. In it I shall try to reflect the current discussion in the Netherlands about skills (drill and practice is a sound, often heard today, especially from university faculty) against the way Euler presented his mathematical expositions.
How did Euler compose his textbooks? How did he think about algebraic and analytical skills? Also the question will pop up whether mathematics is a purely formal system, or whether it should be represent or reflect something real.

We shall do a global reading of the *Complete introduction into algebra* (1770) and also study some fragments from the *Introductio in analysin infinitorum* (1748). These Euler texts I have also used in work with schoolclasses and with mathematics teachers, two activities about which I shall also report. Generally the results were surprising: the experience was stimulating for the students and it confronted the teachers with the fact that much 18th century knowledge has leaked away.

Back to 2-hour Workshops abstracts
Back to the Main Themes

Oral presentations

(Ordered alphabetically)

Name	Title	Theme	Language	Country
Barbazo Eric	Le rôle de l'Association des Professeurs de Mathématiques de l'Enseignement Public (APMEP) et en son sein de Gilbert Walusinski, dans la création des Instituts de Recherche sur l'Enseignement des Mathématiques (IREM). 1955-1975 : 20 années de transformation de l'enseignement des mathématiques en France	5	French	France
Bečvářová Martina	History of Mathematics as a Part of Mathematical Education	3	English	Czech Republic
Bernard Alain	History of science and technology in the French system for teacher training: about a recent initiative.	3	English	France
Blanco Abellán Mónica	The Teaching of Differential Calculus at Military and Engineering Schools in 18th Europe	5	English	Spain
Boettcher Frauke	The "Apollonius' Problem"- the History of its Solutions as Tool for Instruction of Geometrical Methods	1	English	Germany
Cesar de Mattos Marafon Adriana	Process of recognition in the History of Mathematics	4	English	Portugal
Chocholová Michaela	Wilhelm Matzka (1798 – 1891) and his algebraical works	6	English	Czech Republic
Clark Kathleen	Reflection and revision: A first experience with a "Using History in the Teaching of Mathematics" course	2	English	USA
Crilly Tony	Teaching or Research? Cambridge University in the nineteenth century	5	English	UK
De Klerk Johan	History and epistemology as tools in teaching Mathematics	2	English	South Africa
Demetriadou Helen	Didactical and epistemological issues related to τhe concept of proof. Some mathematics teachers' ideas about the role of proof in Greek secondary curriculum	2	English	Greece
D'Enfert Renaud	Du calcul aux mathématiques? L'enseignement mathématique à l'école primaire en France, 1960-1985	5	French	France
Docampo Rey Javier	Manuscripts and teachers of commercial arithmetic in Catalonia (1400-1521)	5	English	Spain
Durand Viviane	La théorie du syllogisme formel d'Aristote : une première rencontre avec les concepts fondamentaux de la sémantique logique.	2	French	France
Gerini Christian	Les Annales de Mathématiques de Gergonne : un journal du 19ème siècle numérisé et médiatisé au bénéfice d'une interdisciplinarité entre mathématiques, histoire, didactique et philosophie.	1	French	France
Giacardi Livia	The Italian School of Algebraic Geometry and the Formative Role of Mathematics in secondary teaching	5	English	Italy

Glaubitz Michael R.	The use of original sources in the classroom. Reading Al-Chwarizmi’s ‘al-jabr’ with 9th-graders. An empirical study.	2	English	Germany
Godard Roger	La programmation linéaire et ses racines	1	French	Canada
Gropp Harald	The relations between mathematics and music in different regions and periods of world history.	4	English	Germany
Horng Wann-Sheng	Studying Indicators of Professional Development: An HPM dimension	3	English	Taiwan
Jancarik Antonin	The Influence of IT on the Development of Mathematics and on the Education of Future Teachers	3	English	Czech Republic
Jahnke Niels	Students working on their own ideas. Bernoulli‘s lectures on the differential calculus (1692) in grade 11	1	English	Germany
Kastanis Andreas	The Teaching of Descriptive Geometry in the Greek Military Academy during 19th Century	5	English	Greece
Kastanis Nikos, Verykaki Katerina	The Conceptual Change Theory as a New Didactical Trend for the History of Mathematics	1	English	Greece
Kiernan James F.	How much history of mathematics must an Early Childhood math teacher know?	3	English	USA
Kvasz Ladislav	Historical and epistemological aspects of teaching algebra	1	English	Slovakia
Lawrence Snezana	Alternatives to Descriptive Geometry – Search for a Perfect Technique of Visualising, Communicating and Teaching Space	5	English	UK
Lepka Karel	C. and. K. Mathematicians Olympics	6	English	Czech Republic
Liu Po-Hung	Investigation Of Student Perceptions Of The Infinity—A Historical Dimension	2	English	Taiwan
March Robert, Sakarovitch Joël	L’enseignement de la géométrie descriptive dans les Ecoles d’ingénieurs en Europe au XIX^{e} siècle	5	French	France
Morey Bernadete Barbosa	Instruments of navigation and teacher training	3	English	Brazil
Métin Frédéric, Guyot Patrick	Erasmus Habermel's geometrical square	6	English	France
Nikolantonakis Constantin	Did we have “Revolutions” in Mathematics? Examples from the History of Mathematics on the light of T. S Kuhn’s historical philosophy of science	1	English	Greece
Petrakis Sifis	The Role of the Fifth Postulate in the Euclidean Construction of Parallels	3	English	Greece
Poulos Andreas	A multidimensional approach of “de L’ Hospital’s rule”	2	English	Greece
Provost Sylvie	Pourquoi “ faire Histoire “ dans l’Industrie, la Recherche et l’Enseignement ?	1	French	France
Rogers Leo	Leonard and Thomas Digges, Sixteenth Century Mathematical Practitioners	4	English	UK
Rowbottom Darrell	Gambling Scenarios and the Foundations of Probability	1	English	UK
Russ Steve	Bernard Bolzano and the Making of Meaning in Mathematics	6	English	UK
Sebastiani Ferreira Eduardo	Ethnomathematic's use in Indian teacher’s formation	4	English	Brazil
Siegmund-Schultze Reinhard	Nazi Rule and Teaching of Mathematics in the Third Reich	6	English	Norway
Silva Maria do Céu	Bento Fernandes’ *Tratado da arte de arismetica* (Porto, 1555)	2	English	Portugal
Sisma Pavel	Teaching at the technical universities in retrospect	5	English	Czech Republic

Siu Man-Keung	Harmonies in Nature : A Dialogue Between Mathematics and Physics	1	English	China
Strantzalos Athanassios	Geometric Transformations as a means for the introduction of interdisciplinarity and of educational elements in High School	1	English	Greece
Swetz Frank	Historical Problems: A Valuable Resource for Mathematics Classroom Instruction	2	English	USA
Tisseron Claude	Enseignements d'histoire, épistémologie et didactique des mathématiques orientés sur l'étude des phénomènes de construction des connaissances	3	French	France
Trkovská Dana	Algebra and Geometry in our countries after the Erlangen and the Merano Programme	6&5	English	Czech Republic
Venegas Leonardo	Prague et l'infini	6	French	Colombia
Volkert Klaus	The Problem of Space in Geometry	3	English	Germany
Wilson Robin	Lewis Carroll In Numberland	4	English	UK
Windsor Will	Analysing the Historical Development of Division May Provide Insights for Improved Teaching of the Concept and Algorithm	2	English	Australia
Yevdokimov Oleksiy	Investigation of high order curves: the way, when history and mathematics come together	3	English	Australia
Zormbala Konstantina	Prospective mathematics teachers' research work on historical records as part of their initial training: A case study in Greece	3	English	Greece

Back to the Main Themes

Oral presentations
ABSTRACTS

(ordered by theme 1, 2, 3, 4, 5, 6)

Theme 1

***Theme* 1**

THE "APOLLONIUS' PROBLEM"- THE HISTORY OF ITS SOLUTIONS AS TOOL FOR INSTRUCTION OF GEOMETRICAL METHODS

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I will present a historical approach for the use in mathematical education during the advanced study period in the program for secondary-school-teacher-training. The aim of my approach is to accentuate and clarify the differencies of geomatrical methods.
The Apollonius' Problem was passed down by Pappus of Alexandria (ca. 290 -- ca. 350) from the lost book *De tactionibus* written by Apollonius of Perga (ca. 262 -- 190). Throughout history the Apollonius' Problem has often served mathematicians to present their mathematical ability and the efficiency of their employed mathematical method. The problem is given as follows:
Given three objects, each of which may be a point, a line, or a circle, draw a circle that is tangent to each.
This problem has an interesting history
As historical starting point I take a learning situation where a female student adresses herself to a master. This situation is given by the correspondence between Elisabeth of Bohemia (1596 – 1632) and René Descartes (1596 – 1650) in the 17th century. In their letters a methodological discussion by solving the Apollonius' Problem takes place
Looking back and forward from the mentioned historical learning situation the history of the problem presents mathematical interests, different geometrical styles, methodological developments and their limitations. Furthermore the historical context of the problem enables to establish connections between mathematics and the social and cultural history often ignored and negated. Due to this multidisciplinary aspects open new perspectives on mathematics and their methods.
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***Theme* 1**

LES ANNALES DE MATHEMATIQUES DE GERGONNE : UN JOURNAL DU 19EME SIECLE NUMERISE ET MEDIATISE AU BENEFICE D'UNE INTERDISCIPLINARITE ENTRE MATHEMATIQUES, HISTOIRE, DIDACTIQUE ET PHILOSOPHIE

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Les premiers grands périodiques consacrés aux mathématiques ont fait l'objet récemment de numérisations permettant de mettre à la disposition des chercheurs et des enseignants des documents historiques pour la discipline et utiles pour son enseignement.
Nous avons conduit à son terme, grâce à un partenariat avec le programme NUMDAM du CNRS, la numérisation du premier de ces grands journaux : les Annales de Mathématiques Pures et Appliquées du mathématicien français Joseph-Diez Gergonne, publiées mensuellement de 1810 à 1832, et diffusées à travers toute l'Europe. .
Parallèlement à ce travail, nous avons exploité des extraits de ce journal dans le cadre de cours de mathématiques en premier cycle universitaire, et dans celui de séminaires d'épistémologie et histoire des sciences pour des étudiants en début de troisième cycle scientifique.
Enfin, nous achevons la réalisation d'un site internet consacré à ce document, et plus largement aux périodiques mathématiques européens du 19ème siècle.
Cet exposé se propose donc :
1/ de montrer les différentes étapes qui ont conduit de l'étude du document original sous tous ses aspects (mathématique, historique, épistémologique, didactique, etc.), à sa numérisation et à sa médiatisation.
2/ d'illustrer par des exemples significatifs l'intérêt didactique du document et de son exploitation dans un cours de mathématiques : émergence du concept de vecteur et représentation géométrique des nombres complexes, évolution du calcul différentiel alors confronté au problème de la validité du concept d'infiniment petit ou de celui de limite, étude d'une démonstration de Galois sur les fractions continues.
3/ de relier de façon transdisciplinaire, comme cela a été fait dans les séminaires d'épistémologie mentionnés plus haut, les mathématiques, la philosophie générale, et l'épistémologie des mathématiques. On voit en effet s'affronter dans ce périodique, au fil des échanges entre les mathématiciens de tous les horizons, et dans des articles de pures mathématiques comme de « philosophie mathématique » -intitulé de l'une des rubriques des Annales de Gergonne-, des courants d'idées et de pensées qui expriment bien ce lien indéfectible, et pourtant bien souvent négligé aujourd'hui dans l'enseignement, entre cette science et la philosophie. Les mathématiques post-révolutionnaires étaient ainsi influencées aussi bien par la métaphysique et le sensualisme néo-condillacien que par le positivisme naissant, et les textes des Annales de Gergonne reflètent concrètement ces influences. On y voit aussi à l'œuvre un débat acharné entre les tenant d'un réalisme géométrique hérité des philosophies classiques et les avancées idéalistes d'un courant davantage nominaliste. Enfin, la philosophie kantienne s'y manifeste de façon inattendue dans des articles de calcul différentiel, et s'y trouve critiquée et rejetée sur la question de la validité du concept d'infinitésimal : la question mathématique est alors abordée dans une véritable réflexion ontologique.
Le recours aux textes originaux sera le fil conducteur de cette démonstration en trois points, afin d'illustrer au mieux ce double intérêt didactique et transdisciplinaire.
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***Theme* 1**

LA PROGRAMMATION LINEAIRE ET SES RACINES

Roger Godard
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La programmation linéaire est un exemple d'application de l'algèbre en éducation, et qui a de nombreuses ramifications en économie et en génie.
Le problème de la programmation linéaire est de minimiser (maximiser) une fonction objective linéaire sujette à un ensemble de contraintes linéaires. Ces contraintes peuvent être converties en contraintes d'égalité en introduisant des variables d'écart. Ces contraintes sont des hyperplans et l'ensemble des solutions forme un polytope convexe. Alors au moins un des sommets du polytope correspondra à la solution optimale. L'aspect géométrique du problème de minimisation est alors facile à souligner.
En 1963, dans son livre «Linear programming and Extensions », Dantzig présenta une table pour retracer les origines de la programmation linéaire avec Fourier en 1823, gauss en 1826, Minkowski en 1896, Farkas en 1903. Notre intention est de faire un balayage sur l'histoire de la programmation linéaire et son aspect interdisciplinaire en utilisant les références données par Dantzig. Ces références sont d'ailleurs très incomplètes, et notre objectif est de présenter une étude plus approfondie notamment sur les contributions de Minkowski, de Farkas et de de la Vallée Poussin en 1912. Rappelons que la programmation linaire permit à Dantzig et à Kantorovich d'obtenir des prix Nobel en Économie.

Plusieurs aspects importants de la programmation linéaire ont été généralement négligés dans les études précédentes sur les origines de la programmation linéaire: (1) l'aspect géométrique et la convexité, (2) le concept de dualité, (3) l'analyse de la sensibilité des variables et des coefficients, (4) le lien avec l'algèbre linéaire et la méthode d'élimination de Gauss, (5) les approches algorithmiques. Pendant les années de 1940, un pas important fut accompli en programmation linéaire par l'introduction du concept de dualité par von Neumann, Gale, Kuhn et Tucker. Dans l'optimisation duale, un problème de maximisation est transformée en un problème de minimisation. Cette découverte, exploitée avec délices au début de la programmation linéaire a des ancêtres très honorés en Mathématiques que nous essaierons de commenter.

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***Theme* 1**

STUDENTS WORKING ON THEIR OWN IDEAS. BERNOULLI'S LECTURES ON THE DIFFERENTIAL CALCULUS (1692) IN GRADE 11

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The paper reports about a teaching sequence in which sections of Johann Bernoulli's Lectures on the differential calculus (1692) are read with students of grade 11. The students try to think themselves into the ideas of mathematicians living at a different time and in a different culture. Doing this they deepen their understanding of the differential calculus and they get get aware more conscientiously of their own ideas on mathematics
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***Theme* 1**

THE CONCEPTUAL CHANGE THEORY AS A NEW DIDACTICAL TREND FOR THE HISTORY OF MATHEMATICS

Nikos Kastanis, Katerina Verykaki
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Ideas, studies and research on the nature, the meaning and the role of conceptual changes within the educational context are widely developing during the last years. Didactics of mathematics could not stay indifferent to that trend.
So it becomes very interesting to illuminate their influence and to point out its didactic significance as well as the didactic role of history of mathematics in that cognitive context.
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***Theme* 1**

HISTORICAL AND EPISTEMOLOGICAL ASPECTS OF TEACHING ALGEBRA

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In my presentation I would like to discuss the educational implications of an epistemological reconstruction of the historical development of algebra, which was published as (Kvasz 2006). This reconstruction is based on Wittgenstein's picture theory of meaning. In the history of algebra I was able to distinguish six developmental stages. These stages differ in their fundamental epistemological structure. To illustrate these differences let us compare the different ways each of these stages conceives of a solution of algebraic equations. To solve an equation means:

1. To find a ***regula***, i.e. a rule written in ordinary language, which makes it possible to ***calculate*** the „thing", that is, the root of the equation (Cardano 1545).

2. To find a ***formula***, i.e. an expression of the symbolic language, which makes it possible to ***express*** the root of the equation in terms of its coefficients, the four arithmetical operations and root extraction (Viete 1591).

3. To find a ***factorization*** of the polynomial form, i.e. to ***represent*** the polynomial form as a product of linear factors (Descartes 1637).
4. To find a ***resolvent***, i.e. to ***reduce*** the given problem by means of a suitable substitution, to an auxiliary problem of a lesser degree (Euler 1770).
5. To find a ***splitting field***, i.e. to ***construct*** the field that contains all the roots of the equation (Gauss 1799).
6. To find a ***factorization*** of the Galois group of the splitting field, i.e. to ***decompose*** the group of automorphisms of the field into blocks (Galois 1832).

In my presentation I would like to discuss the epistemological differences of these stages and the implications of these differences for mathematics education.

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***Theme* 1**

DID WE HAVE "REVOLUTIONS" IN MATHEMATICS? EXAMPLES FROM THE HISTORY OF MATHEMATICS ON THE LIGHT OF T. S KUHN'S HISTORICAL PHILOSOPHY OF SCIENCE

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The second half of the 20th century witnessed a kind of revolution in the history and philosophy of science with the edition of Thomas S. Kuhn's book *Structure of Scientific Revolutions,* published in 1962, which view of science is generally labeled "historical philosophy of science".

In my presentation I will try to argue whether or not the "historical philosophy of science" can be applied to mathematical truth. My intention in considering the problem whether or not Kuhn's view of scientific revolutions is applicable to mathematical truth in the following lines has been inspired by my study on the formation of Euclidean and non-Euclidean geometry.

Our understanding of mathematics should be based on the concept of mathematics as a quasi-empirical science.

Generally mathematicians believe that "Mathematical truth is always true and is not time dependent". Mathematics acquired the first paradigm from ancient Greece and was one the first sciences to employ deductive inference and characterized by the axiomatic method. Mathematics is always related to the ideal world, as distinct from the real world through the procedure of abstraction. Mathematics in Islam, Latin Middle Ages and Renaissance were more or less influenced by these aspects. But, these characteristics are not a priori but historical acquisitions.

We can say, by following the statements of Imre Lakatos, that mathematics is normatively pure deductive science but it is quasi-empirical.

We assume that a mathematical proposition is true if its postulates are true. But the postulates and the standards of rigor are time-dependent, p.e. the mathematical terms of Hilbert's *Grundlagen der Geometrie* are not identical to Euclid's. Mathematical theories must be quasi-empirical and time dependent. We can see that mathematical theories are not so empirical as in the natural sciences, not much restrained empirically by the real natural and life world. With mathematical revolutions, the new theory must become fashionable and the older theory may become unfashionable, but has not been totally discarded. For example, the Non-Euclidean geometry became very influential and much used by mathematicians but the truth-values of the Euclidean geometry were preserved, fact that proves that revolutions in mathematics are not so radical as in the natural sciences. In mathematics, with revolutionary transformations losses of the former theory are not so much and not so destructive, and its properties are usually inherited through translations of mathematical languages or by shifting the modes of mathematical thought. Mathematical knowledge develops contentually and institutionally and also gradually and revolutionarily.

With the formation of non-Euclidean geometry, the truth-value of Euclidean geometry was kept in a certain sense, but not totally. Euclid's geometry with the parallel postulate in ancient Greece had a different meaning after the publication of David Hilbert's *Grundlagen der Geometrie* of 1899. We are going to affirm Kuhn's observation quoted "revolutionary changes involve discoveries that cannot be accommodated within the concepts in use before they were made" can be applied to the history of mathematics.

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***Theme* 1**

POURQUOI " FAIRE HISTOIRE " DANS L'INDUSTRIE, LA RECHERCHE ET L'ENSEIGNEMENT ?

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Dans les domaines d'activité scientifique comme la recherche, l'enseignement ou l'industrie, je montrerai l'importance d'y exercer un recul historique :

- avec le physicien W. H. Bragg (1862-1942), dans sa recherche d'adéquation des modèles théoriques, au réel contradictoire, lors de la conférence qu'il fit à Dundee en 1912 ;
- avec Paul Langevin (1872-1946), à la conférence donnée au musée pédagogique en 1926, où il présente « la valeur éducative de l'histoire des sciences » ;
- enfin, avec les ingénieurs Combes, Phillips et Collignon, qui montrent les progrès de la mécanique appliquée, au travers d'une étude historique de cette discipline, pour l'exposition universelle de 1867 à Paris.

NB : Présentation avec transparents en anglais

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***Theme* 1**

GAMBLING SCENARIOS AND THE FOUNDATIONS OF PROBABILITY

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This paper will explore how a consideration of gambling scenarios can be used in order to provide an engaging basis for learning about probability, while drawing both on history and epistemology. It will also discuss how such an approach can help to ensure that teachers and learners do not fall into one of the most serious pitfalls when it comes to understanding probability: failing to appreciate that it is Janus-faced, and can be interpreted in radically different ways – either *aleatory* or *epistemic* – depending on context. The basic premise is that a problem-led approach is often ideal for introducing people to new mathematical notions.
First, the paper will provide a brief overview of how probability theory was developed in order to solve concrete gambling problems. It will then consider some of the illuminating difficulties encountered by its architects, in providing a philosophical foundation for the mathematics, and explain their pedagogical significance. In particular it will discuss the question of whether the probability calculus can be used to reflect chances in some objective sense, or just how one should reason when uncertain, which is still a live topic in contemporary philosophy.
Second, it will show how a consideration of good betting behaviour can be used to derive the axioms of probability in a neat and memorable fashion, according to the popular *epistemic* view that probabilities are rational degrees of belief.

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***Theme* 1**

HARMONIES IN NATURE: A DIALOGUE BETWEEN MATHEMATICS AND PHYSICS

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The usual practice of teaching mathematics and physics as two separate subjects certainly has its grounds. However, such a practice deprives students of the opportunity to see how the two subjects are intimately interwoven.
In collaboration with colleagues in Department of Physics the author has tried to design a short enrichment course for students who possess the motivation and the calibre for mathematics and physics, and who are about to embark on their undergraduate study. The proposed series of lectures/tutorials attempts to integrate the two subjects with a historical perspective. Such a short course with the title *Harmonies in Nature : A Dialogue Between Mathematics and Physics*, which lasts for thirty hours spaced out in ten sessions, had a trial run in the Spring of 2006. One may detect a degree of resemblance of this title to the 1618 treatise of Johannes Kepler (*Harmonice mundi*) and a hint of the 1638 treatise of Galileo Galilei (*Discorsi e dimonstrazioni matematiche intorno a due nuove scienze*). Much as the idea of conducting the lectures in the form of a dialogue is innovative and attractive, such an implementation is also extremely demanding and difficult to be carried out in a reasonably satisfactory fashion. The word "dialogue" used here is only to indicate the intended integration between the two subjects.
The underlying theme would be the role and evolution of mathematics (mainly calculus, with related topics in linear algebra and geometry) in understanding the physical world, from the era of Isaac Newton to that of James Clerk Maxwell and beyond, to that of Albert Einstein. In other words it tries to tell the story of triumph in mathematics and physics over the past four centuries.
Ideas and methods devised by ancient Greeks and ancient Chinese on problems in quadrature are brought in to contrast the power of calculus developed during the seventeenth/eighteenth centuries, culminating in the Fundamental Theorem of Calculus with its generalized form (Stokes' Theorem) established through the

development of theory of electromagnetism. In connection with the notion of waves, the idea of differential equations introduces trigonometric (and inverse trigonometric) functions and exponential/logarithmic functions in a light not usually emphasized in a traditional school curriculum in mathematics. Geometry comes in when the theory of relativity is explored, and probability theory comes in when quantum mechanics is explored, even though both topics can only be treated after a fashion, considering the level of the class.
The physics provides both the sources of motivation and the applications. Along the way both ideas and methods appear, to be learnt in an interactive manner through discussion, homework assignments and tutorials.
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***Theme* 1**

GEOMETRIC TRANSFORMATIONS AS A MEANS FOR THE INTRODUCTION OF INTERDISCIPLINARITY AND OF EDUCATIONAL ELEMENTS IN HIGH SCHOOL

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Introduction Indications for the role of the History of Mathematics in their Didactics with specializations concerning the Geometric Transformations.
The main educational and interdisciplinary scopes and the corresponding preferability of the theoretical framework of the Theory of Transformations.
1. A brief survey for the introduction of the needed theoretical background of the transformations of the Plane Euclidean Geometry, reducing the formalities with the aid of simple figures.
Here, the History of Geometrical Transformations plays an essential role in choosing the basic notions and pointing out their functionality.
2. Some examples of geometrical exercises for the training of pupils in "globally thinking", which is the main educational purpose.
3. Towards the interdisciplinarity: we shall briefly explain examples for the use of (not necessarily Euclidean, but conceivable by the pupils) transformations and of the corresponding methodology
(a) in producing physical laws, and in indicating the basics of the Special Theory of Relativity for the 1-dimensional space (therefore 2-dimensional space-time), and
(b) in explaining some of the characteristics of Escher's engravings, mainly as concerns the analogous of the infinite strip on the Euclidean Plane in the Hyperbolic case, after a descriptive introduction of Poincaré's disc model for the Hyperbolic Geometry, and the production of the needed knowledge of the geometric theory of circles in the plane.
Both (a) and (b) reflect historical facts, that will, thus, be directly embedded in the course. Especially (b) has its historical roots in what Escher himself pointed out about the help provided for him after he learned elements of Hyperbolic Geometry through Coxeter; our proposal is that a corresponding historical note should be embodied in the course.
4. Indications of certain details and extensions.

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Theme 2

***Theme* 2**

REFLECTION AND REVISION: A FIRST EXPERIENCE WITH A "USING HISTORY IN THE TEACHING OF MATHEMATICS" COURSE

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In this oral presentation I will share results from a study conducted at Florida State University (FSU) in which I analyzed students' experiences with the capstone project in the course, "Using History in the Teaching of Mathematics." The course, required of both undergraduate and graduate mathematics education majors, has in recent years at FSU been structured as a survey course, with a biography paper for a final project. The course I designed focused on presenting various middle school and high school topics from an historical perspective, while emphasizing essential mathematics and pedagogy related to the topics.
The focus of the study was to investigate how pre-service mathematics education students draw upon their experiences with various course activities to consider a topic (or collection of related topics) historically and subsequently develop a teaching unit or model lesson (the capstone project in the course) for use in future secondary mathematics teaching. In the capstone project, students were required to examine their topic along several dimensions. For example, the teaching unit might ideally include cultural and humanistic influences and historical texts and problems. Data sources included the project, presentation, and accompanying resource file, as well as student journal reflections documenting their historical, mathematical, and pedagogical progress during the course. In addition to providing feedback on the students' teaching unit and presentation, I evaluated the student projects using the journal reflections they provided. Lastly, I used my own weekly reflections on course

activities, as well as the evaluation of the quality of the capstone projects produced, to consider potential course revision for future course offerings.
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***Theme* 2**

HISTORY AND EPISTEMOLOGY AS TOOLS IN TEACHING MATHEMATICS

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In a previous presentation in the ESU series (page 181 of the Proceedings of a Conference on History and Pedagogy of Mathematics, Uppsala, 2004) it was argued that Mathematics should not be viewed as an independent, separate subject field only, but rather as a wider field fitting into a framework of contexts. Some of the contexts mentioned were those of history, science, society, nature and religion. Viewed in this way, one has, as teacher and lecturer of Mathematics, the opportunity of stressing the cultural embeddedness of mathematics in societal life.

In the present discussion, this topic is developed further. Two matters will specifically be addressed, namely (a) the role of the history of Mathematics and Technology, and (b) the role of the epistemology of Mathematics in class discussions.

The role of the history of Mathematics and Technology in class discussions: Presently, it is widely argued that it is to a Mathematics student's benefit if the history of Mathematics could be integrated as a tool in the teaching of Mathematics, and –even broader – of the Sciences. Unfortunately, there is a negative aspect to this matter that has to be mentioned, namely the poor standing of History (both as a subject field in its own right and as a general background tool to class discussions) due to the view of most students and teachers. Kauffman (1991:185) remarks with respect to his subject, Chemistry, which can certainly be generalised to include any other natural science subject: "In short, most students of Chemistry, in common with their instructors, have only minimal interest in, or knowledge of, the history of Chemistry". Therefore, in my Mathematics classes I have embarked on a road of not only discussing aspects of the history of Mathematics, but also of discussing some historical aspects of technology. I have found that students are much more motivated when history and course material are related in this way.

The role of the epistemology of Mathematics in class discussions: While the cultural embeddedness of Mathematics could be emphasised very well by using the history of the subject as background, it could be stressed still further by adding some aspects of the epistemology of Mathematics (and again broader, of the Sciences). The general public (students included) tend to regard scientific theories as "eternal truths". Seeing that this cannot be true in the Sciences, I have decided that this aspect needs some elucidation in my classes. In this respect, Hooykaas remarks (1999:94): "Not all that is 'scientific' is necessarily true; and not all that is 'true' is 'scientific'!" In this conference discussion, attention will be paid to some related matters, for instance that a mathematical theory in Applied Mathematics never equals reality or nature (but that it gives, at most, a description of reality via the process of model building).

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***Theme* 2**

DIDACTICAL AND EPISTEMOLOGICAL ISSUES RELATED TO THE CONCEPT OF PROOF. SOME MATHEMATICS TEACHERS' IDEAS ABOUT THE ROLE OF PROOF IN GREEK SECONDARY CURRICULUM

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This presentation first examines some epistemological issues concerning the teaching, understanding and performing demonstrations. Such issues are: (a) the necessity of using proofs, (b) the difference between logical justification and the immediate appreciation of the self-evident properties of geometric figures, (c) the different epistemological meanings of proof connected, either with incomplete argumentations - which however lead to obvious results – or, with the requirement of not-contradicting an axiomatic system, which is finally persuasive.

The paper continues with a study about the role of proof in the Greek secondary curriculum, and examines the opinions of mathematics teachers about the necessity of teaching demonstrations. Because of the pressure due to a huge mathematical content to be taught, especially in the upper Greek secondary education (ages 16-18), leads to abandoning many proofs both in Analysis and Geometry. This situation causes an interesting disagreement amongst the mathematics teachers' community: some believe that the main function of proof is the development

of rational thinking, while others argue that the use of too many and too difficult proofs cause problems in understanding and learning mathematics.
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***Theme* 2**

LA THEORIE DU SYLLOGISME FORMEL D'ARISTOTE : UNE PREMIERE RENCONTRE AVEC LES CONCEPTS FONDAMENTAUX DE LA SEMANTIQUE LOGIQUE

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Comme de nombreux logiciens contemporains le reconnaissent, les intuitions géniales d'Aristote lui ont permis de construire un système formel appuyé sur les concepts fondamentaux de la moderne sémantique logique : classification et modélisation des énoncés de la langue ordinaire par des énoncés formels, au sens où ils sont caractérisés par leur forme: énoncés singuliers ; énoncés généraux universels ou particuliers; définition des propositions comme entités linguistiques susceptibles de porter le vrai ou le faux; introduction de lettres de termes pour caractériser la forme des énoncés et construire les figures du syllogisme; notion d'interprétation des énoncés formels ; notion de validité logique des syllogisme concluants; distinction entre vérité de facto et vérité nécessaire obtenue comme conséquence d'un syllogisme concluant à prémisse vraie; mise en œuvre conjointe de méthodes syntaxique et sémantique pour établir soit la validité d'un syllogisme à partir des syllogismes posés comme concluants a priori ; soit sa non validité en produisant un contre exemple, c'est à dire une interprétation des lettres de termes pour lesquels les prémisses sont vraies tandis que la conclusion est fausse.
On a souvent reproché à Aristote, à juste titre, d'avoir proposé un système clos et pauvre au regard des besoins logiques pour les mathématiques. Mais ici, dans la perspective didactique qui est la nôtre, cette faiblesse va apparaître comme un atout. En effet, le système proposé par Aristote, par sa simplicité, par le nombre fini des énoncés considérés, permet de proposer à des étudiants de deuxième ou de troisième année d'université scientifique une première rencontre avec ces concepts fondamentaux développés en logique mathématiques dans une perspective articulant syntaxe et sémantique depuis les travaux pionniers de Frege, Russell et surtout Wittgenstein et Tarski.
Dans cette communication, je développerai dans une première partie, les éléments permettant d'affirmer que les concepts fondamentaux de la sémantique logique sont déjà présent chez Aristote en m'appuyant sur les livres II (De l'Interprétation) et III (les Premiers analytiques) de l'Organon, dans la traduction due à Jean Tricot, ainsi que sur els analyses de Patzig (1988).
Dans une seconde partie, je présenterai les modalités de travail développées depuis plusieurs années avec des étudiants de deuxième et troisième années d'université pour leur permettre, à partir de l'étude guidée de quelques textes primaires tirés des livres II et III de l'Organon, de rencontrer, le plus souvent pour la première fois, ces notions fondamentales. Ceci jouant le rôle de prolégomènes avant d'aborder les textes des logiciens de la fin du XIX° siècle et du début du XX° siècle.

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Mots clés: histoire et philosophie de la logique – didactique des mathématique – sémantique logique – théorie du syllogisme formel
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***Theme* 2**

THE USE OF ORIGINAL SOURCES IN THE CLASSROOM. READING AL-CHWARIZMI'S 'AL-JABR' WITH 9TH-GRADERS. AN EMPIRICAL STUDY

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Occasional or frequent historico-mathematical excursions from the straight and traditional school syllabi are by many believed to provide valuable enhancements to the mathematical and cultural literacy of learners. In particular, reading original sources is assumed to possess promising potential as a motivational and cognitive instrument. Yet rather little is known *empirically* about the actual effectiveness and possible drawbacks of historico-mathematical teaching, let alone reading sources in class. Hence in a comparative study, several hundred ordinary German 9thgraders were taught quadratic equations with and without paying substantial regards to the historical origin and development of today's thinking and solving schemes. The historically enriched teaching unit included amongst other things the reading of parts of Al-Khwarizmis 'al- jabr' (~820

A.D.) and accordingly pursued a widely unfamiliar but promising hermeneutic approach to the subject. In this contribution I am going to present, compare and discuss the conceptual design, parts of the realization and some major results of the analysis of either unit.
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***Theme* 2**

INVESTIGATION OF STUDENT PERCEPTIONS OF THE INFINITY—A HISTORICAL DIMENSION

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Infinity is a significant element for understanding calculus, yet school teaching and educational studies consistently suggest that its counter-intuitive nature confused students. Before 19th century, mathematicians in history heavily relied on intuition to deal with the concept of infinity. However, these intuitive approaches usually yield conflicting conclusions. The concept of infinity, as Fischbein, Tirosh, and Hess (1979) indicated, involves contradictory nature, which is arisen from our experiential logic of finiteness. These inconsistent phenomena prompted Aristotle to distinguish between potential infinity, an endless dynamic process, and actual infinity, a static and completed object, and exclude the use of actual infinity in mathematical domains. Such a distinction and argument, nonetheless, is an impractical attempt for professional mathematicians.

Several studies (e.g., Tsamir & Dreyfus, 2002) have indicated that the difficulty that students encounter while dealing with the concepts of infinity is similar to those that mathematicians faced in history. This purpose of this study was to investigate college students' perceptions of paradoxes of the infinity and observe how their perspectives shifted back and forth while facing contradictory phenomena. Three different questionnaires (1. comparing cardinalities of infinite sets; 2. conflicting results of divergent series; 3. Zeno's paradoxes) were conducted to observe students perceptions of infinity and this study was interested in looking at how these college students resolved contradicted conclusions that they made. Tsamir and Drefus (2002) indicated four common approaches that students were likely to use: (1) seeing infinity as a single entity (all infinite sets are equal); (2) comparing the size of infinite sets by observing from which subset more and longer intervals have been omitted; (3) considering a set that is strictly included in another set has fewer elements than that other set (i.e., part-whole relationship); (4) treating infinite sets as incomparable. This study generally supports Tsamir and Drefus's findings. Tsamir and Drefus further noted students usually exhibited no particular tendency to use one-to-one correspondence and another study conducted by Waldegg (2005) also claimed, as compared to Cantor's one-to-one correspondence for establishing his theory of infinity, Bolzano's criterion, based on the part-whole relationship, is more intuitively acceptable by students because it is closer to concrete experience. Nonetheless, my study yielded different results.

My study not only suggests future research of this line should pay attention to the dialectical process of students' discourse to detect their core beliefs about infinity, but also analyses students' conflicting perceptions on a history basis. This is an interdisciplinary attempt to take HPM (History and Pedagogy of Mathematics) and PME (Psychology of Mathematics Education) issues into account. It is hoped the findings may benefit both academic fields and elicit more attention on this aspect.
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***Theme* 2**

A MULTIDIMENSIONAL APPROACH OF "DE L' HOSPITAL'S RULE"

Andreas Poulos
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We shall present an experimental approach in the teaching of de l' Hospital's Rule which was carried out during a course of lectures on Differential Calculus given to students of age 16- 17 which expressed some special interest in Mathematics among those studying in the Experimental School of the University of Macedonia, at Thessaloniki, Greece. After a typical presentation of de l' Hospital's Rule and the teaching of typical exercises concerning the computation of indeterminate forms using limiting procedures, the students were encouraged to see the subject from different perspectives. They "read" in a naive way the original text of de l' Hospital's book *Analyse des infiniment petits* [1696] ACL-Editions, Paris, 1988) having been given the information that this was the first textbook in Analysis. This reading led to interesting discussions, as students were impressed by the exclusively geometrical style of this book and the fact that there were no derivatives in the text, but only differentials. The students were even more surprised when they realised through their reading of the History of Mathematics, some "strange", unexpected events, e.g., that the so-called "de l' Hospital's Rule" was not a discovery of the Marquis de l' Hospital. In this way it has become obvious that a typical kind of lesson can bring out diverse, interesting problems and questions: historical, ethical, mathematical, naive epistemological, didactical, political, editorial, etc.

Students were asked to attempt to write biographies about the Marquis de l' Hospital and members of the

Bernoulli family including main events of that historical period, especially events related to the development of Calculus. Additionally, they were encouraged to sketch and find other intuitive proofs of the Rule. They came in contact with other indeterminate forms, such as $1^{\infty}, \infty^{0}, \infty - \infty$, etc and their history. Using Taylor series gave them another methodological view of the computation of limits. The students found many and different kinds of information about de l' Hospital's Rule in the Internet, they developed all of these and they are currently writing a pamphlet about the multidimensional approaches to de l' Hospital's Rule in the History and teaching of Mathematics.
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***Theme* 2**

BENTO FERNANDES' *TRATADO DA ARTE DE ARISMETICA* (PORTO, 1555)

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The principal aim of this talk is to present some details of the study of problems in the *Tratado da arte de arismetica*, written by Bento Fernandes and published in Porto, in 1555. As it is the first treatise of a Portuguese author that has come down to us, and where algebra is included, it deserves special attention since it constitutes a testimony of the state of development of algebra in Portugal, in the middle of 16th century. As we know, Pacioli's *Summa* was, at the time, the most influential mathematical text, so we can ask if it was the source of the algebraic material of Bento Fernandes'. To answer this question, we did a comparative study between the *Tratado da arte de arismetica*, the *Summa*, and other abacus' books from the 13th to 15th centuries. We present here some conclusions of that study.
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***Theme* 2**

HISTORICAL PROBLEMS: A VALUABLE RESOURCE FOR MATHEMATICS CLASSROOM INSTRUCTION

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"Where can I find some good problems to use in my classroom?" is a question often asked by mathematics teachers. The answer is simple "The history of mathematics".

Since earliest times, written records of mathematical instruction have almost always included problems for the reader to solve. The luxury of a written discourse and speculation on the theory of mathematics appeared fairly late in the historical period with the rise of Greek science. Records from older civilizations: Babylonia, Egypt and China, reveal that mathematics instruction was usually incorporated into a list of problems whose solution scheme was then given. Quite simply, the earliest known mathematics instruction concerned problem solving-the doing of mathematics. Obviously, such problems, as the primary source of instruction, were carefully chosen by their authors both to be useful and to demonstrate the state of their mathematical art.' The utility of these problems was based on the immediate needs of the societies in question and thus reflect aspects of daily life seldom recognized in formal history books. Such collections of problems are not limited to ancient societies but have appeared regularly throughout the history of mathematics.

In the literature of mathematics, thousands of problems have been amassed and await as a ready reservoir for classroom exercises and assignments. The use of actual historical problems not only helps to demonstrate problem solving strategies and sharpen mathematical skills, but also:

- imparts a sense of the continuity of mathematical concerns over the ages as the same problem or type of problem can often be found and appreciated in diverse societies at different periods of time;
- illustrates the evolution of solution processes -- the way we solve a problem may well be worth comparing with the original solution process, and
- supplies historical and cultural insights of the peoples and times involved.

This talk will discuss the use of historical problems in classroom situations. References will be made to specific problems and problem sequences.
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***Theme* 2**

ANALYSING THE HISTORICAL DEVELOPMENT OF DIVISION MAY PROVIDE INSIGHTS FOR IMPROVED TEACHING OF THE CONCEPT AND ALGORITHM

Will Windsor
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The evaluation of historical literature has been used rarely to support the theory that a conceptual understanding of mathematics improves learner outcomes. The presentation will investigate how the historical development of division has influenced modern teaching strategies and ultimately children's understanding of the division concept and algorithm. It will explore historical evidence to verify the notion that a conceptual understanding for elementary students is paramount in developing an understanding of the division concept.

The presentation will examine how many of the rotely taught procedures used in modern classes have been developed out of the language and materials used from the past, with many of these procedures having been taken out of their cultural contexts and taught to students without reference to their historical background. By analysing historical data, reasons concerning the persistent misunderstandings of the division concept may be illuminated upon. Furthermore, by examining the progression and sequential development of different algorithms, an insight into conceptual frameworks utilised in various societies becomes evident. The investigation will highlight the conceptual framework that may allow students the best opportunity to fully master division.

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Theme 3

Theme 3

HISTORY OF MATHEMATICS AS A PART OF MATHEMATICAL EDUCATION

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At present, our education system faces quite a number of serious problems. Besides the financial ones which can be solved neither by teachers nor pupils, there are many issues which have been discussed for a long time. Let us mention some of them: a fitting lessons structure, pupils' knowledge standard at all types of schools, a uniform "state leaving exam", uniform admission exams for the same types of university studies, overloading pupils, humanization of education, discipline of both pupils and teachers, new didactic methods, etc.

At present, at the time of the fast development in all areas of human life, our university graduates are usually fully qualified people but very often they lack the minimum universal insight. They lack an ability to formulate their ideas both verbally and in writing, they are hardly able to express their suggestions of improvement or inventions, they lack an ability to communicate as they can hardly express their ideas even in their mother tongue. Already teachers at secondary schools face this problem; but to make matters worse, so do teachers at universities. Where does the problem originate? This is not only the problem of pupils who of course can also be blamed (indifference or narrow specialisation). A certain part of guilt also lies with teachers. Because how many of them submit unaided seminar essays, request analyses of issues studied, critical studies of the given problems etc.? How many teachers urge students to do unaided creative work? Probably not many of them as such activities are both time consuming and require a lot of expert knowledge. There has been a demand for both good preparation work and successive marking and results analysis.

The contribution will focus on the knowledge and results obtained during the last ten years at the Czech Technical University in Prague and Charles University with teaching special courses on the history of mathematics and on the development of mathematical thinking within the education of future technicians and future secondary teachers. The role of the history of mathematics in their education, their motivation for studies and the similar as well as different aspects of their preparations will be discussed.

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Theme 3

HISTORY OF SCIENCE AND TECHNOLOGY IN THE FRENCH SYSTEM FOR TEACHER TRAINING: ABOUT A RECENT INITIATIVE

Alain Bernard

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For a long time, the French education ministry has shown a great interest by introducing history of sciences in education. This recently became evident in secondary schools through official instructions that specify various learning subjects for which an historical approach is recommanded. In the case of mathematics, much work has already been done on this subject for the past three decades within the framework of the IREM (Institut de Recherche sur l'Enseignement des Mathématiques). But since such a framework does not exist for all sciences, many teachers of science still have some difficulties to insert this point of view in their lessons or just ignore the work that has been already done on this. In some cases they have undertaken something but have no real contact with a community, which may promote their efforts. The same is true for the many persons involved in teacher training: even the IREM groups usually depend on the math department of each university and are not directly implanted within the teacher training institutions, namely the so-called IUFM (Institut Universitaire de Formation des Maîtres).

For all these reasons, the ReForEHST group was constituted last year between several IUFM in order to favor exchanges within a community of teachers, trainers and researchers, to develop new tools to help teachers and

promote research (this acronym means ***Re**cherche et **For**mation en **E**pistémologie et **H**istoire des **S**ciences et des **T**echniques*, that is *Research and Training in Epistemology and History of Sciences and Techniques*).
The group promotes reflections focusing on the following key issues:

- Which history of sciences should be taught, why should it be taught?
- Which school subject matters could be concerned and what are the connections with sciences education?
- What should be the main avenues in training teachers in a positive way to promote the history of sciences?

Our aim is to focus on those questions in order to specify the practical aspects of the official instructions. Also, we suggest various orientations in teacher training. Particularly, we propose to emphasize the contribution of history of science in the following set of themes:

- the structural knowledge of the educational system,
- the place of science in society;
- Science and citizenship;
- Science and language;
- Methodology of sciences (what is a well argued-speech? What is rationality?…),

We will describe the activities already undertaken by the group (two colloquium, a web site) and in which way it means to tackle the issues mentioned above.
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***Theme* 3**

STUDYING INDICATORS OF PROFESSIONAL DEVELOPMENT: AN HPM DIMENSION

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In this presentation, I am going to report some first-phase consequence of my project, "Studying Indicators of Professional Development: An HPM Dimension" (funded by NSC, August, 2005 – July, 2008). Assuming that the HPM can enhance math teachers' pedagogical content knowledge, my project aims at dealing with the following three problems:

(1) Once an ideal teacher has competence in HPM, what specific aspects of PCK in his / her teaching practice can be identified?

(2) How an ideal teacher can be benefited from the HPM as he or she becomes a mentor for student teacher and novice teacher?

(3) How can we transform an ideal teacher's HPM competencies into indicators for QA (quality assurance) and Mentoring?

To the end of the first year, I will have collected data of teachers' reflection on a questionnaire in which half the questions are about how teachers integrate law of cosine and different versions of proof for Pythagorean Theorem into their classroom. In addition, they are also requested to comment on a real teaching project implemented by a senior high school teacher.
I will try to analyze the hundred copies of questionnaire in order to understand whether the teachers are able to integrate history into teaching in a significant way, at least in their initial stage of designing teaching project. This may well suggest various ways of improving teacher's PCK in terms of HPM.
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***Theme* 3**

THE INFLUENCE OF IT ON THE DEVELOPMENT OF MATHEMATICS AND ON THE EDUCATION OF FUTURE TEACHERS

Antonin Jancarik
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The ever growing availability of Information technology (IT) and its increasing performance possibilities influence more or less every field of science, including mathematics. The aim of this paper is to illustrate how the use of computers influenced the development of mathematics, especially in the second half of the 20 th century, and how these changes reflect in the education of future mathematics The influence of computers can be observed in the following areas:

New fields of mathematics emerge

The development in IT allowed for new fields to emerge and develop. All of these are to a certain extent derivatives of IT: computability and complexity, theory of formal languages and automata and borderline disciplines (like cryptology or cryptography).

Mathematical Analysis developments

Mathematical analysis was mainly influenced by the CAS (Computer Algebra Systems) and graphic systems. Thanks to CAS, completely new calculation methods were derived, for example the method for finding primitive functions. These methods are designed exactly according to specific computer Graphic systems enable fast and elegant function rendering, including functions with several variables, and thus offer new approaches to their study.

Geometry developments

The use of computer geometry systems like, for example, the Cabri system allowed for a completely new type of exercise to emerge, mainly thanks to the movement tools. At the same time, geometry education began to be tackled from a different point of view.

Algebra developments

In algebra, combinatorics, theory of numbers or logic, the main role of computers used to be to verify (e.g. The Four Color Problem), to construct examples and counter examples. However, since the late 1980s computers have also been used to automatically prove theorems in predicate logic of the first order - for example, Robbinson hypothesis. It is the possibility to prove mathematical assertions automatically that gives a completely new role to IT in mathematics.

The Influence on the Education of Future Teachers

The changes taking place along with the increased use of IT in mathematics have an influence on the education of future mathematics teachers. On the one hand, computers allow for fast solutions to some classical problems, on the other hand, they enable us to study some completely new problems. How this change of environment can be utilized in the education of

future teachers will be demonstrated on an example of the Faculty of Education, Charles University in Prague, and on the difference between the use of IT in the old and the new accredited study courses.

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***Theme* 3**

HOW MUCH HISTORY OF MATHEMATICS MUST AN EARLY CHILDHOOD MATH TEACHER KNOW?

James F. Kiernan

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This question has been asked over and over by students in the program at Brooklyn College. Some teachers who are receiving Masters Degrees in Early Childhood Education with a specialization in mathematics are under the false impression that they need only know as much mathematics as in contained in the grade level they plan to teach. They take a course in the history of mathematics expecting that it will be all history and very little mathematics.

The course that has been created at Brooklyn College is designed to give an overview of the history of mathematics, including basic ideas in calculus and modern algebras and geometries. This course is challenging but rewarding to the students who take it. The students, also, get some appreciation of a historical approach to the subject. It is important that they do not view this subject as a collection of fables.

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***Theme* 3**

INSTRUMENTS OF NAVIGATION AND TEACHER TRAINING

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Trigonometry and its history has always been present in our studies. Our last work was about the application of trigonometry to the Portuguese navigations in the XV-XVII centuries.

The theme is important to us Brazilians because the arrival of the Portuguese in 1500 started the events that determined our national identity. Also, it is a recurrent theme in social studies, specifically in the History of Brazil, but there we never meet any mention of mathematics. To the undergraduate math students in our courses, the information that the Portuguese navigations were enterprises that were undertaken with help of mathematics is very interesting. Thus, there is no lack of interest and motivation in the beginning of our courses.

While we talk about the details of the enterprise and try to create a living picture of what a journey across the sea to America or to India was in that time, attention levels remain high. It is interesting to see what instruments were used and how they actually helped locate the ship in the middle of the ocean.

The difficulties begin at the moment when we start to detail the mathematical knowledge that was necessary to construct and use these instruments. In the case of circular instruments like the quadrant and the astrolabe, the necessary knowledge is no more than angles, their measure and a few simple properties. For linear instruments, like the kamal, the Jacob staff or the cross staff, however, the necessary mathematics includes understanding and manipulating trigonometric tables.

It is at this moment, therefore, when we start to stress the mathematics behind the instruments, that the students begin to loose interest and have difficulty in accompanying the course. That situation worries us because we are working with (present or future) math teachers, so we decided to investigate the situation more deeply. Our objective is thus to identify the reasons that cause the students to loose interest and to determine how to overcome the linked difficulties that arise in the course.

Our preliminary data suggest that the difficulties are mostly due to the lack of knowledge about trigonometric concepts and of the use of trigonometric tables. Students who were interviewed indicate that trigonometry,

although a standard school subject, is very poorly studied in the high schools and avoided thereafter whenever possible.
There remains the question of how to capitalize on the students' initial motivation and interest in order to overcome their rejection of trigonometry. This is the next step of our research.
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***Theme* 3**

THE ROLE OF THE FIFTH POSTULATE IN THE EUCLIDEAN CONSTRUCTION OF PARALLELS

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The central theme of the workshop I plan to organise is the presentation of a new (constructive) interpretation of Euclid's Fifth Postulate and some of its epistemological consequences.
The standard interpretation of the Fifth Postulate, from Proclus up to our days, is non-constructive (i.e., the Fifth Postulate has nothing to do with the parallel construction) and mainly a uniqueness interpretation (i.e., the Fifth Postulate guarantees the uniqueness of the constructed parallel line). This traditional approach will be criticised and it will be explained why it turns out to be unsatisfactory.
More than that, it will be shown that the Fifth Postulate (used for the first time in Proposition I.29 of Euclid's Elements) serves a certain constructive purpose, essential for the legitimacy of the parallel construction in Proposition I.31. Within our interpretation the place of the parallel construction after the first use of the Fifth Postulate is a necessity and not a curiosity (as it is in the standard interpretation).
Our interpretation (which, as far as I know is new in the literature) is based on some very general principles regarding construction, K(a, P), of an object a satisfying a geometric property P. In that way the parallel construction plays a key role in our understanding of the Euclidean constructions. The demonstration of the epistemological role of the Euclidean constructions is, in our view, also important to the teacher of mathematics, since basic geometric constructions are studied even in high school.
Closely connected to the construction K(a, P) is the concept of existence, $\exists aP(a)$, of an object a satisfying a geometric property P. We shall examine the relation of these two concepts in Euclid and compare them to Hilbert's axiomatic approach to Euclidean geometry. We shall exhibit the main differences between Euclid and Hilbert on existence and construction and the important consequences of these differences on the philosophy of geometry.
Though the study of the connections between Euclidean and non-Euclidean geometries is a large enterprise, we shall discuss the Bolyai construction of limiting parallels, showing that it is not a legitimate construction from the Euclidean point of view. Further discussion of Lobachevsky's axiom (the analogue to the Fifth Postulate in hyperbolic geometry) through Pejas' classification theorem of Hilbert planes, will show that Euclidean geometry is not that easily comparable to hyperbolic geometry as the standard, axiomatic approach indicates.
Moreover, our analysis of the concepts of existence and construction can also be connected to the foundations of mathematics. Brouwer's "discrete" development of the continuum (intuitionism), Hilbert's symbolic finitism (formalism), Frege's logicism, Poincaré's conventionalism, to mention only some great examples, are all considerably influenced by the evolution of non-Euclidean geometries in the 19th century. Their views on existence and construction of mathematical concepts can be related to the Euclidean paradigm.
Finally, we shall provide suggestions for further research on the subject, which springs from this new interpretation of Euclidean geometric constructions

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***Theme* 3**

ENSEIGNEMENTS D'HISTOIRE, EPISTEMOLOGIE ET DIDACTIQUE DES MATHEMATIQUES ORIENTES SUR L'ETUDE DES PHENOMENES DE CONSTRUCTION DES CONNAISSANCES

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Mots clés: enseignement universitaire, histoire et philosophie des mathématiques – didactique des mathématique – formation des maîtres

Dans l'insertion professionnelle, la formation des maîtres occupe une place importante à l'université. Cette place a augmenté avec les contraintes d'adaptation du système éducatif aux nouveaux publics et dispositifs. Un texte officiel de 1998 précise que "*L'évolution du dispositif actuel de formation continue des enseignants doit répondre à plusieurs objectifs :*
-lier étroitement la formation initiale et la formation continue de façon à assurer la continuité entre cursus universitaire et professionnalisation ; mieux inscrire ainsi dans le temps l'acquisition progressive des compétences professionnelles complexes et multiples requises pour l'exercice du métier". (Directions de l'enseignement scolaire et de l'enseignement supérieur (12 mars 1998)). Anticipant ce texte, et à partir de notre expérience de formation des maîtres en France (IREM) et à l'étranger, nous avons conçu et réalisé depuis 1993 divers modules optionnels d'enseignements d'histoire, épistémologie et didactique des mathématiques. La raison d'être de ces modules est qu'un important travail de formation des enseignants nous paraissait à conduire sur deux niveaux : un niveau est relatif aux connaissances et techniques permettant *l'organisation* de situation d'apprentissage correspondant aux souhaits des programmes et adaptées aux nouveaux publics (par exemple « études de textes, apprentissage de la démarche scientifique, recherches de problèmes et études de situations complexes »)., Un autre niveau de formation est relatif aux *conceptions des mathématiques et des conditions de leur apprentissage* permettant aux enseignants de conduire avec succès de telles situations. La forte nécessité d'un travail sur le second niveau, particulièrement sur les conceptions de mathématiques, est la principale motivation de ces modules. Nous pensons que ce travail doit commencer au cours de la formation initiale universitaire.

Aujourd'hui, les modules de 3ème année et 4ème année ont été intégrés aux nouveau cursus LMD et mastère. L'Université de Lyon I a aussi développé depuis plus de dix ans des modules analogues dans d'autres disciplines scientifiques. Pour en rester aux mathématiques, mais il en est de même pour les autres disciplines, ces modules ont en commun l'objectif de constituer un ensemble cohérent en proposant en seconde, troisième et quatrième année d'université des enseignements d'*initiation* à l'histoire et à l'épistémologie des mathématiques, puis à leur didactique pour répondre à la demande des étudiants en général et aux besoins de pré-professionalisation des enseignants d'autre part. Cette pré-professionnalisation étant destinée d'une part à donner l'idée que les mathématiques se *construisent,* d'autre part à permettre aux futurs enseignants de compléter leur formation dans un domaine où leur culture de base est insuffisante comme, par exemple, la géométrie élémentaire.

Il s'agit ainsi de permettre aux étudiants qui le souhaitent d'élargir leur compréhension de leur discipline par une approche historique, épistémologique et didactique, L'intégration d'une approche historique de la discipline complète l'approche dominante de type "axiomatique-déductiviste". L'approche didactique étant ancrée et motivée par l'approche historique et épistémologique. La durée des modules a varié de 30 à 60h annuelles. La plus grosse partie du travail est constituée d'études guidées de documents en petits groupes. Ces documents sont constitués d'extraits de textes mathématiques historiques, voire de textes sur les mathématiques. Leur étude comporte une dimension mathématique dans laquelle les connaissances des étudiants doivent être retravaillées pour s'adapter aux nouveaux contextes d'étude. L'évaluation se réalise généralement par un mémoire en petits groupes.

L'expose présentera le dispositif global et les variations suivant les modules, un aperçu des contenus et des méthodes, ainsi qu'une évaluation basée d'une part sur des réactions ou travaux d'étudiants, d'autre part sur les résultats d'un mémoire de DEA réalisé en 2004 sur une partie d'un module en 4ème année, concernant l'étude de l'analyse.

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***Theme* 3**

THE PROBLEM OF SPACE IN GEOMETRY

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In my talk I will discuss a decisive period in the development of solid geometry: its revival around 1800, in particular with Legendre's "Elements of Geometry" (1794), and its systematic construction during the 19^{th} century, in particular the progressive installation of rigor.
Solid geometry is in my opinion an important theme (cf. Chr. Zeeman: Three-dimensional theorems for schools [The Mathematical Gazette March 2005]) often neglected in the teaching of mathematics. This is due at least to two reasons: firstly, solid geometry is more difficult than plane geometry, and, second, it is often neglected in the teaching of future teachers as well. An historical perspective may help to overcome these difficulties. For that reason I will also try to integrate some non-mathematical features important to the development of solid geometry in my talk (e.g. some problems of crystallography).
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***Theme* 3**

INVESTIGATION OF HIGH ORDER CURVES: THE WAY, WHEN HISTORY AND MATHEMATICS COME TOGETHER

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We would like to pay attention to pedagogical effect of using well-known books of the beginning of the twentieth century for teaching contemporary mathematics courses on undergraduate level: Loria, "Spezielle algebraische u. transscendente ebene Kurven", 1902; Teixeira "Traite des courbes speciales remarquables planes et gauches", 1908-1909; Wieleitner, "Theorie der ebenen algebraischen Kurven hoherer Ordnung", 1905; "Spezielle ebene Kurven", 1908. Shortly characterising the importance of historical context in teaching mathematics courses, we would like to note that from a constructivist perspective it is easier for a student, under appropriate arrangement of teaching, to act as an architect, to reveal the truth and construct new knowledge, comparing it with the findings, which had been done by the famous mathematicians long before.
We will focus on some topics from Advanced Calculus, first of all, theory of high order curves in the plane. For conducting students' activities teachers have to posses the knowledge about these books. It can be achieved through integration some material of such old invaluable mathematical books in teaching process. We will show examples how to use brilliant material of these books in a classroom in the scope of corresponding historical-mathematical environment. We will trace the links between different curves from the historical point of view. We will consider different properties of the curves through their historical significance and didactical value. It is interesting to note that studying these books teachers become "learners" together with the real learners. In particular, we analyse their influence on teachers' understanding the role of history and didactical implementations to the learning process. We hope that using such material in teaching process will give opportunity for teachers as well as students to grasp the idea of evolutional development of high order curves throughout the centuries. We will present a flexible structure of units for students' collaborative and individual work in a classroom using one-component and multi-component tasks, which are powerful didactical tools for teachers in their practice work. As one-component task we call the one, which aimed at students' inquiry work for finding properties of a certain high order curve in the plane, i.e. students have a clear formulated direction for their activities in a classroom. As multi-component task we call the one, which aimed at students' inquiry work for finding properties of different high order curves connected with each other, i.e. students investigate curves without clear indication the direction for their activities in a classroom: what properties and for what curves are to be suggested. We are going to consider Newton parabola punctata, Descartes curve, Steiner hypocycloid, Doppel-Herz curve and many others. Some of them are famous and well-known, others are forgotten or almost unknown.
We would like to emphasize that historical dimension in teaching mathematics has invaluable importance. In our opinion, it is a kind of innovative approach – to learn and teach new mathematical content through constructing new knowledge from old books, which serve as bridges between nowadays and past.
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***Theme* 3**

PROSPECTIVE MATHEMATICS TEACHERS' RESEARCH WORK ON HISTORICAL RECORDS AS PART OF THEIR INITIAL TRAINING: A CASE STUDY IN GREECE

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Research on the historical development of the mathematical education has been conducted in many countries during the last decades. This research is considered by the educational and mathematical communities to be a basic tool for analysis and interpretation of today's status of the school subject of mathematics. In parallel, in the educational circles, the question of how the history of mathematics relates to its teaching and learning has been posed. The issue of the use of original historical mathematical texts as means of education of both the pupils and the prospective mathematics teachers (p.m.t.) has come to the fore front.

Our research, which is conducted with prospective mathematics teachers, can be placed within this framework, and aims at widening the above area and relating the teaching of mathematics to the history of mathematical education. Means for this constitute the original historical records which are looked for, elaborated and studied by the students at the History Archive of the place where they study. The activity has been conducted with students at the Department of Mathematics of the University of the Aegean (Samos) and has taken place at the History Archive of the Prefecture of Samos. In relation with the research, several thousands manuscripts, which are relevant to the education in Samos in the 19th century, have been studied.

The present talk will describe the above activity, will discuss its consequences for the p.m.t. and will propose other alternative ways of education for the p.m.t. with the aid of original historical records on the history of mathematical education.

The talk will show that the particular activity combines teaching with research, helps the p.m.t. to shape a perception about the mathematical education of their native land and urges the p.m.t. to embody historical facts from the mathematical education into the teaching of mathematics. However, the main feature of this activity is the perception that is formed, namely that the history of the mathematical education is not merely a subject of study, but also a topic for research. The activity aims at converting the p.m.t. from passive recipients into active producers of knowledge.

Back to oral presentations abstracts

Theme 4

***Theme* 4**

PROCESS OF RECOGNITION IN THE HISTORY OF MATHEMATICS

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I have found support in the archives to say that mathematical ideas are constructed under political contexts. It is not new, philosophers like M. Foucault, F.W Nietzsche used to disagree that there is a fix point where we can find the ‘pure’ knowledge. U. D’Ambrosio (2005) uses the term ‘co-opt’ to designate the “...strategy to organize a society and to legitimate a power structure”. The power of scientific community is based on acceptance of the members on the authority (Weber, 1978) of committees that are established by this specific community.

I considered it sufficient to study the historical recognition process of Cayley´s work, in order to ‘to locate’ the ‘political ground’ or ‘opinion ground’ on mathematical ideas.

The working mathematicians who had a special position in the History of Mathematics called our interest in investigating the process of historical recognition. Thus, firstly, examining the mechanisms of recognition in Mathematics, and secondly it is essential studying Arthur Cayley's work (1821-1895).

In fact, I have no interest to value the historical period or a mathematician. I decided to study nineteenth century because, firstly, it is a period not so far from our epoch, and secondly it is enough far to give the guarantee about Cayley’s name in the History of Mathematics.

I decided to look for mathematicians, who did not have their income as mathematicians as it was, at that time, very complicate for someone to belong to the community. I focused on Arthur Cayley and Sylvester. As Sylvester lived for a long time in the USA, I would need to go to the USA, which would be very difficult as staying in the UK. For this reasons I decided to focus only on Arthur Cayley.

Mathematical historians believe that Cayley developed some of his best work when his income was not related to his research in Mathematics; it had been present in the community by means of publications. What is interesting is that although he did not have a direct connection with the Mathematics Community in terms of his income, he belonged to this community with his publications. The following years, in 1863, Cayley was appointed as Professor of Pure Mathematics at the University of Cambridge.

The focus of this study is on the recognition mechanisms, therefore it is reasonable based on the archives to make a historical interpretation about the ‘genius’ and the ‘knowledge’ in Mathematics.

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Theme **4**

THE RELATIONS BETWEEN MATHEMATICS AND MUSIC IN DIFFERENT REGIONS AND PERIODS OF WORLD HISTORY

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The relation between mathematics and music (and astronomy) will be described and discussed. In medieval Europe the socalled "quadrivium" collected arithmetic, geometry, astronomy, and music and their relations into a basic education for learned scholars. Arithmetic and geometry as the two pillars of early mathematics are combined with music via the most ancient and important application of mathematics, the science of astronomy.

However, these traditions are much older than medieval Europe having their origin in the Ancient Near East several millennia earlier. In Mesopotamia there was a close connection between the so-called religious and the so-called scientific sphere. In such an environment of cosmological and theological beliefs the science of astronomy was pushed forward in order to support these relations between the human world below and the divine world above. Both arithmetic and geometry connected and supported these ideas and were developed to a high level.

Later this knowledge was brought from the East into the Greek and Roman world. This Greek context of mathematics and music and astronomy is much better known to the modern world and connected to names like Pythagoras and Ptolemaios and to terms like numerology, intervals, harmony etc.

Again several centuries later the Islamic world collected the cultural ideas of its predecessors and neighbours and created a new context of mathematics and music (and astronomy). Greek ideas as well as Indian, Persian, and Mesopotamian concepts were spread to regions as far as Andalusia and Central Asia.

Via different routes the Arab knowledge entered medieval Europe (see above). The quadrivium with all its traditions was certainly very influential, at least until the seventeenth century. Maybe Kepler, mainly in Praha, was the last European scholar who was fully aware of this ancient connection and transformed it into the beginnings of modern European science.

This talk will try to focus on those aspects which are not so well known and only partially investigated. On the one hand, the relations between mathematics, music, and astronomy belong to the key parts of the evolution of human culture. On the other hand, our modern partition of science into many subdisciplines and the growing specialization as well as the gap between mathematics and astronomy on one side and cultural history on the other side makes research in this area difficult.

In the triangle of relations between mathematics, music, and astronomy the focus will be on the mathematics - music edge. In this sense some episodes of this long history will be presented in order to give a more detailed view on the general historical aspects. If time allows, a short discussion of the reception of these historical relations in the last centuries in Europe will conclude my talk.

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Theme **4**

LEONARD AND THOMAS DIGGES, SIXTEENTH CENTURY MATHEMATICAL PRACTITIONERS

Leo Rogers

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Leonard Digges published ‘Prognostication Everlasting’ in 1555, and opens it with an *apologia* ‘Against the reprovers of astronomy and science mathematical’ where he states that ‘the ingenious, learned and well experienced circumspect student mathematical receiveth daily in his witty practices more pleasant joy of mind than all thy goods (how rich soever thou be) can at any time purchase.’

This belief in the spiritual as well as the practical benefits of mathematics had been put forward earlier by Robert Recorde, and more forcefully by John Dee in his ‘Mathematicall Praeface’ (1570), and was to be the guiding theme for the rising group of mathematical practitioners who sought to instruct artisans and others in the practices ans applications of mathematics.

In this vein, in 1556 Leonard Digges’s ‘Tectonicon’, a practical manual, advertised itself as ‘most conducible for surveyors, landmeters, joiners, carpenters and masons’. Here, the new ‘mathematical practitioners’ are seen to be defining themselves and competing with mechanicians and artisans. This competition was a recurrent problem during the 16th and 17th century in England.

Leonard’s son Thomas (1546–1595) received his early education from his father but he died when Thomas was fourteen and was looked after by a famous family friend, John Dee. Thomas received advanced mathematical instruction from Dee and collaborated with him in various mathematical and astronomical works, where he was a champion of Copernicanism, Thomas also completed a number of his father’s unfinished manuscripts.

This paper is a continuation of my work first published in HPM 2004 and explores the relationship between Dee and the Digges family, and the social and economic context of mathematical instruction and the development of the applications of mathematics in England in the latter sixteenth century.

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***Theme* 4**

ETHNOMATHEMATIC'S USE IN INDIAN TEACHER'S FORMATION

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My work regarding the Indigenous Education has more than 20 years, always with the purpose of forming the Researcher Indian teachers, inside the Etnomathematics, that is, to be Indian teacher, the ethnograph of its culture and builder of this knowledge with the western Mathematics, in order to propose to your students an educational process with criteria. It is the Etnomathematic Research Program proposal created by Ubiratan D´Ambrósio.
With the Waimiri-Atroari, of Amazonas' North, I have been working for 10 years with the purpose to form the village Researcher teacher. I present some of the mathematical knowledges of Waimiri-Atroari and, mostly, any field researches done by them and how these researches were transformed in mathematical activities, that were used in the village classes.
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***Theme* 4**

LEWIS CARROLL IN NUMBERLAND

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Charles Dodgson (Lewis Carroll) is best known for his Alice books, 'Alice's Adventures in Wonderland' and 'Through the Looking Glass'. But he was a mathematics lecturer at Christ Church, Oxford University. What mathematics did he do? How good was he? In this presentation I shall outline his work in geometry, logic, algebra, and recreational mathematics.

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Theme 5

***Theme* 5**

LE ROLE DE *L'ASSOCIATION DES PROFESSEURS DE MATHEMATIQUES DE L'ENSEIGNEMENT PUBLIC* (APMEP) ET EN SON SEIN DE GILBERT WALUSINSKI, DANS LA CREATION DES IREM; 1955-1975: 20 ANNEES DE TRANSFORMATION DE L'ENSEIGNEMENT DES MATHEMATIQUES EN FRANCE

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L'A.P.M.E.P., depuis 1910.

1) Une association de professeurs de l'enseignement secondaire jusqu'en 1945.
2) Une modification de la sociologie des adhérents après 1945.

- Qui permet à l'association de se développer dans les milieux universitaires nationaux et internationaux.
- Qui place l'association en interlocuteur auprès du ministère.

Les présidences de Gilbert WALUSINSKI et André REVUZ.

1) Gilbert WALUSINSKI, une action de longue haleine, entre 1955 et 1980.

- Il prône, dans un premier temps, la réforme de l'enseignement et de la formation pour palier au manque d'ingénieurs, de techniciens et de professeurs.
- Il développe des conférences régulières sur les mathématiques dites modernes en liaison avec la SMF et retranscrites dans les Bulletins de l'APMEP.

- Il oriente son esprit réformiste vers une réflexion davantage didactique et pédagogique : les mathématiques modernes deviennent un outil de réforme incontournable et humaniste.
- Il contribue à la découverte des expériences belges (PAPY), anglaises (FLETCHER) et canadiennes (DIENES).

2) André REVUZ, l'homme de la réforme des mathématiques modernes.

- Il permet à l'association de développer des liens dans les milieux universitaires.
- Les cours de André REVUZ sur les structures algébriques (espaces vectoriels, groupes, anneaux, corps) sont largement diffusés par l'A.P.M.E.P. et contribuent au développement de l'esprit réformiste.

Une action politique de l'A.P.M.E.P ; qui trouve enfin l'écoute ministérielle.

1) Différentes commissions sont créées par l'association depuis 1957. A l'initiative de Gilbert WALUSINSKI, elles contribuent au développement des mathématiques modernes.

- Commission de réforme du 18 octobre 1956
- Conférence générale sur la formation et le recrutement en 1960.
- Grande Commission de l'APMEP en 1964.
- Commission Recherche et Réforme en 1966.

2) Le travail de la commission ministérielle présidée par André LICHNEROWICZ.

- Les similitudes avec la commission Recherche et Réforme.
- Les premières propositions d'IREM.

3) Edgar FAURE, le ministre qui crée les IREM.

- Le discours à l'assemblée nationale en octobre 1968 qui fait référence à la charte de Chambéry de l'APMEP.
- Le mouvement de mai 1968 qui sert de catalyseur.
- La réforme de l'université d'Edgar FAURE permet la création des trois premiers IREM.

Les IREM créés.

1) Leur lettre de mission et l'absence de statuts. Des moyens de fonctionnement très importants.
2) Entre outil de recyclage et outil de recherche pédagogique.
3) La naissance de la didactique de Guy BROUSSEAU à l'IREM de Bordeaux.

Comment est-on arrivé à enseigner les mathématiques modernes ?

1) L'évolution du concept de vecteur. Du segment orienté à l'élément de l'espace vectoriel.
2) La disparition de la géométrie et son traitement par l'algèbre dans les programmes de 1970 : conséquences du programmes d'Erlangen de KLEIN et du mouvement formaliste de HILBERT ?
3) L'enseignement des structures mathématiques très tôt

- Pour mieux aborder les différentes notions de nombres.
- Pour conceptualiser les infinis dénombrables et continus.

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***Theme* 5**

THE TEACHING OF DIFFERENTIAL CALCULUS AT MILITARY AND ENGINEERING SCHOOLS IN 18TH EUROPE

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Schubring [1] supports the comparative textbook analysis as a means to determine the differences among countries with regard to style, meaning and epistemology, because they are determined by the structures and values of a specific national educational system. The aim of this paper is to compare and analyze the teaching of differential calculus within the military and engineering educational system through the examination of some textbooks written in 18th century Europe. The textbooks which have been analysed to the purpose are: 1) *The Doctrine and Application of Fluxions* (1750) by Thomas Simpson, 2) *Principj di analisi sublime* (1759) by Giuseppe Luigi Lagrange, 3) *Anfangsgründe der Analysis des Unendlichen* (1770) by Georg Friedrich Tempelhoff and 4) *Cours de mathématiques à l'usage du corps de l'artillerie* (1799-1800) by Étienne Bézout. This paper first describes how the textbooks analysed laid the foundations of calculus and then assesses the influence each of these foundational approaches exerted on the resolution of problems and applications, such as the determination of tangents and extreme values.

In 1743 Simpson became a teacher of mathematics at the Royal Academy of Woolwich. In his text, he did not discuss largely on foundations, but he offered a wide range of applications. Although Lagrange and Tempelhoff addressed their textbooks to the students of artillery schools of Turin and Prussia, respectively, they were both concerned with foundations. Actually, Lagrange relied on Euler's epistemological views, thus adopting an innovative point of view. Surprisingly, Bézout relied largely upon Leibniz's epistemological views on the subject and his text is similar to that of L'Hôpital's, written a century earlier. He was connected to the military field, too. In fact he developed his activity at the *écoles militaire*s. His exposition is rather elementary since he did not regard a rigorous exposition of foundations as an essential topic for an engineer's education. Yet his text is included in his volume on Mechanics and Hydrostatics, where the differential and integral calculus play a relevant role.

[1] Bézout, É. (1799) *Cours de mathématiques à l'usage du Corps de l'Artillerie*, III. Paris

[2] Lagrange, J. L. (1759) *Principj Analisi Sublime*. Turin. In Borgato, M. T.-Pepe, L. (1987) "Lagrange a Torino (1750-1759) e le sue lezioni inedite nelle Reale Scuole di Artiglieria", *Bollettino di Storia delle Scienze Matematiche*, II

[3] Schubring, G. (1996) "Changing cultural and epistemological views on mathematics and different institutional contexts in nineteenth-century Europe", in Goldstein et al. (eds.), *Mathematical Europe. Myth, History, Identity*. Éditions de la Maison des sciences de l'homme, Paris, 363-388

[4] Simpson, T. (1750) *The Doctrine and Application of Fluxions*. Printed by J. Nourse. London

[5] Tempelhoff, G. F. (1770) *Anfangsgründe der Analysis des Unendlichen*. Berlin

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***Theme* 5**

TEACHING OR RESEARCH? CAMBRIDGE UNIVERSITY IN THE NINETEENTH CENTURY

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Until the end of the nineteenth century, Cambridge University was the centre for mathematics in the United Kingdom. Apart from the eminent practitioners connected with this university, the university housed the famous Mathematical Tripos, a focus of mathematical education for several centuries. An outline of its history will be given, focussing on the various periods of its reform. Issues raised will include the relationship which existed between educational ideals espoused by the Tripos and the gradual movement towards the university becoming a place for mathematical research.

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***Theme* 5**

DU CALCUL AUX MATHEMATIQUES? L'ENSEIGNEMENT MATHEMATIQUE A L'ECOLE PRIMAIRE EN FRANCE, 1960-1985

Renaud d 'Enfert

IUFM de l'académie de Versailles

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En France, l'enseignement mathématique dispensé à l'école primaire est l'objet d'un fort renouvellement qui commence dans les années 1960 et qui est confirmé avec la réforme dite des « mathématiques modernes » en 1970. La démocratisation de l'accès à l'enseignement secondaire, qui modifie en profondeur la fonction même de l'école primaire, d'une part, et la volonté de rénovation de la discipline elle-même, depuis la maternelle jusqu'à l'université, d'autre part, conduisent à reconfigurer un champ disciplinaire jusqu'alors principalement centré sur des pratiques opératoires renvoyant à la vie quotidienne ou professionnelle. Cette communication se propose d'examiner les principaux enjeux des transformations de contenus et de méthodes qui sont opérés dans les décennies 1960-1970. On s'appuiera à cet effet sur les premiers résultats d'une recherche collective portant sur les grandes réformes de l'enseignement effectuées en France dans cette période.

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***Theme* 5**

MANUSCRIPTS AND TEACHERS OF COMMERCIAL ARITHMETIC IN CATALONIA (1400-1521)

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The growing complexity of commercial practice in late Medieval Europe made knowledge of arithmetic more necessary for many people, particularly for merchants. The teaching of the Indo-Arabic numeration system, its methods of calculation and applications to commerce was made mainly through vernacular treatises that started to appear towards the end of the thirteenth century.

During the last 40 years, an increasing number of commercial arithmetic and algebra texts from the period 1300 – 1600 have been studied. Considering the great economic development of many Italian cities during this period, it is hardly surprising that most of these texts were composed in Italy. Francesc Santcliment's *Summa de l'art d'Aritmètica* is a book on commercial arithmetic that was written in Catalan and published in 1482 in Barcelona. It was the first mathematics book printed in the Iberian Peninsula and the second printed commercial arithmetic in Europe. However, even when printed treatises like this started to be published and to be widely used, manuscripts continued to play an essential role in the teaching of commercial arithmetic and algebra. In fact, some of these manuscripts give us a closer view on the teacher's daily work, since they show how contents of the printed treatises were adapted to each educational context.

This talk is based on my research on Catalan manuscript sources for commercial arithmetic and algebra during

the late Medieval and early Renaissance periods. I will use three of these manuscripts to show which contents were studied and which teaching methods were used, among other topics.
The first of these three sources is a collection (ca. 1445) of more than 200 unsolved exercises that are systematically ordered, most of them dealing with exchange and calculation of prices. The second one is a treatise that follows, at least partially, the "practice and doctrine" of Galceran Altimir, who taught arithmetic in Barcelona at least from 1460 onwards. This treatise, which has important coincidences with Santcliment's *Summa*, appears in a handbook for merchants (ca. 1490).
The latest manuscript (ca. 1520) has a far more complex content than the others. A good part of it contains a series of notes translating and adapting parts from Luca Pacioli's *Summa de Arithmetica, Geometria, Proportioni et Propotionalità* (first published in Venice in 1494). Circumstantial evidence suggests that its author, who was mainly interested in the algebra chapters of Pacioli's treatise, is the Majorcan Joan Ventallol. As far as I now, this manuscript contains the first known algebra treatment in a vernacular Iberian language.
Connections between these texts and French-Provencal and Italian mathematics of the time are obvious, but we also find some characteristic features, like the use of a certain kind of diagrams for equations to clarify the steps in solving problems algebraically in the last manuscript.
The contents of commercial arithmetic have changed little since then, and these works are the predecessors of modern elementary arithmetic texts. Furthermore, they played an essential role in the transmission and development of Arabic algebra.
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***Theme* 5**

THE ITALIAN SCHOOL OF ALGEBRAIC GEOMETRY AND THE FORMATIVE ROLE OF MATHEMATICS IN SECONDARY TEACHING

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The Italian school of algebraic geometry came to be in Turin at the end of the nineteenth century, under the guidance of Corrado Segre. It soon brought forth such significant results that it came to represent a leading light ("führende Stellung") at an international level, as F. Meyer and H. Mohrmann note in the *Encyclopädie der mathematischen Wissenschaften*. The most illustrious of its members included, to name but a few, Gino Fano, Beppo Levi, Guido Castelnuovo, Federigo Enriques, Francesco Severi, Alessandro Terracini and Eugenio Togliatti.
The great significance of the scientific results obtained by the school has led many to forget, or at best to attach only secondary importance to the mathematics teaching related issues which occupied many of its members, including Segre himself, his academic associate Gino Loria and, above all, his disciples Castelnuovo, Enriques and Severi throughout their lives.
An examination of the articles and of other works by these authors dedicated to problems pertaining to teaching, together with the manuscripts of university lectures and a number of published and unpublished letters, reveals a clearly-defined vision of mathematics teaching, directly opposed to that which was inspired by and founded upon the principles of the Peano school. It springs, on one hand, from the Italian geometers' contact with Felix Klein and his important organisational role in transforming mathematics teaching in secondary and higher education and, on the other, from the way in which the authors themselves conceived of advanced scientific research.
The methodological assumptions, which underpin this conception of education and its aims, can be roughly summarised as follows. They believed that teaching should be an active process and develop the students' capacity to discover things for themselves. They sought to bridge the gap between mathematics and all natural sciences in order to make science teaching more interesting and more in touch with the real world. They maintained that logical reasoning and intuition were two inseparable aspects of the same process, and it was therefore necessary for teachers to find the correct balance between the two, moving by degrees from the concrete to the abstract. Finally, they considered that higher mathematics, considered in the context of its historical development, allowed for a better understanding of certain aspects of elementary mathematics, and should consequently have a key role in teacher training.
In my paper, I will illustrate the reasons which led Italian geometers to become so concerned with problems pertaining to mathematics teaching, the epistemological vision by which they were inspired, the various ways in which this interest manifested itself (school legislation, teacher training, text books, university lectures, publications, participation in national and international commissions, etc.), and the influence of Klein's ideas and of other international initiatives on education.

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***Theme* 5**

THE TEACHING OF DESCRIPTIVE GEOMETRY IN THE GREEK MILITARY ACADEMY DURING 19TH CENTURY

Andreas Kastanis

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Greece came into contact with Descriptive Geometry relatively early. Monge's courses were taught for a long time in the Greek Military Academy. During the second half of the 19th century the teaching of Descriptive Geometry blossomed. In some of its applications, and especially in Perspective, there were some ideological implications, which were, however, exceeded. As a base for the Greek handbooks the respective French, written by Leroy or Olivier, were used. During the last two decades of the 19th Century the first books which were published in Greek, were mostly translations by French ones. It is also noteworthy that the vast majority of the professors were military officers.

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***Theme* 5**

ALTERNATIVES TO DESCRIPTIVE GEOMETRY – SEARCH FOR A PERFECT TECHNIQUE OF VISUALISING, COMMUNICATING AND TEACHING SPACE

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History of Descriptive Geometry in France and its utilisation in the French educational system since the 18th century, has already been well documented in the work of Taton (1951), and more recently Sakarovitch (1989, 1995). The history of the technique in England, however, makes a captivating story, particularly as it relates not only to the technique itself, or how the treatises relating to it were translated into English, but because it was also closely related to the establishment of the architectural and engineering professions in Britain.

I will look at the alternatives to Descriptive Geometry in the techniques that appeared in the treatises published during first half of the 19th century in both France and England. I will also trace the history of Descriptive Geometry from its translation into English at the beginning of the 19th century to the end of that century and show that it did find a place in the educational system of English architects, engineers and even mathematicians, but in a modified form. I will also show how descriptive geometry developed in Britain during the 19th century and how textbooks were written on the technique, to be used in some newly established universities. I will examine the differences between the 'original' system of Descriptive Geometry, and the modified, 'anglicized' version of the technique.

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***Theme* 5**

L'ENSEIGNEMENT DE LA GEOMETRIE DESCRIPTIVE DANS LES ECOLES D'INGENIEURS EN EUROPE AU XIX^E SIECLE

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L'objectif de cette communication sera de montrer que l'enseignement de la géométrie descriptive au XIX^e siècle est un excellent laboratoire d'analyse du statut d'une branche des mathématiques appliquées. Les rapports de la géométrie descriptive avec les mathématiques pures comme avec les techniques corrélées seront analysés, afin d'expliciter son rôle dans la formation des élites au moment de l'industrialisation naissante de l'Europe continentale.

On montrera en particulier l'adéquation, au moment de la création de la discipline par Gaspard Monge dans la première Ecole polytechnique, entre la géométrie descriptive et une certaine conception de la formation des ingénieurs civils et militaires. Entre géométrie pure et géométrie appliquée, la géométrie mongienne est porteuse d'une utopie révolutionnaire de réconciliation du théorique et du pratique, du penser et du faire. Cette situation peut expliquer qu'une discipline jusqu'alors inconnue devienne la colonne vertébrale de la nouvelle Ecole polytechnique. Il s'agit aussi pour Monge, de « fabriquer de l'enseignable », pour reprendre une formule d'André Chervel, dans une situation exacerbée par « l'urgence révolutionnaire ».

Le rôle des cours de Gaspard Monge dans le renouveau des études géométriques en France sera également analysé, tout comme l'échec relatif, au début du XIX^{e} siècle, de ses conceptions quant à la formation des ingénieurs. Derrière cet échec se profilent en fait les difficultés récurrentes du système scolaire français à réaliser un lieu d'intégration entre les enseignements techniques et les enseignements théoriques. A l'Ecole des ponts et chaussées, le conflit Brisson/Navier qui sous-tend deux conceptions différentes de la formation des ingénieurs, se noue autour de l'enseignement de la géométrie de Monge. A l'Ecole centrale des arts et manufactures, fondée en 1829, l'importance accordée à la géométrie descriptive dans le curriculum des élèves, marque la volonté des enseignants-fondateurs de renouer avec l'esprit de la première Ecole polytechnique.
En comparant l'évolution, durant la seconde moitié du XIX^{e} siècle, de l'enseignement de la géométrie descriptive dans différentes écoles d'ingénieurs civils et militaires, en France comme en Europe, nous montrerons et les dérives théoriciennes dont elle est l'objet au cours du siècle et les tentatives toujours recommencées d'ancrer cette discipline dans l'enseignement technique.
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***Theme* 5**

TEACHING AT THE TECHNICAL UNIVERSITIES IN RETROSPECT

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In the lecture devoted to the history of teaching at technical universities, we shall focus on the aspects of the personality of a mathematics teacher. It will be shown how the requirements set upon the teachers of these schools developed in connection with the changing content of teaching mathematical subjects and with the rise in the number of students. We shall consider the preparation of mathematics teachers and whether their professional orientation corresponded with the needs of technical universities.

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Theme 6

***Theme* 6**

WILHELM MATZKA (1798 – 1891) AND HIS ALGEBRAICAL WORKS

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Wilhelm Matzka (1798 – 1891) was a Czech regular profesor of mathematics. He was born in Lipertice in Moravia, studied at the secondary school in Chomutov and then at the University of Prague. Then he served many years as an artilleryman by Austria army. At the same time he continued in his education at University of Vienna. His pedagogical activities are adherent to Vienna (1831 – 1837, school for artillery-men), Tarnov (1837 – 1849, philosophical school) and Prague (1849 – 1850, the Czech Technical University; 1850 – 1868, the Prague University). For many years he functioned at the Royal Bohemian Learned Society.
Expertly he attended especially to algebra (algebraic equations, theory of determinants). Then he was engaged in analytic geometry, infinitesimal calculus (differential and integral calculus with geometrical applications), trigonometry and some parts of mathematical physics. In these areas he wrote textbooks, special articles and studies, historical, methodical and popular works.
In the second part of 19th century it was hardly developed study of some parts of algebra, high attention was attended to the theory of determinants. This problem was verey favourite in Czech countries. A lot of less or more original special works came into being. The first books of the theory of determinants, methodical and popular articles were written. Wilhelm Matzka processed the theory of determinants in studies "Principles the theory of determinants" which was published in 1877/8 in publication the Royal Bohemian Learned Society. It will be interesting to compare today's and that time textbooks of algebra, in which were interesting and nearly forgotten aplications of determinants.
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***Theme* 6**

C. AND. K. MATHEMATICIANS OLYMPICS

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There was an enormous industrial growth in Bohemia and Moravia in the second half of the 19th century. That was the reason why technically educated experts were needed. Besides the traditional educational system there was another interesting form of study, that enabled gifted students to improve und enhance their knowledge.

The organisation of specialists in mathematics called ”Jednota českých mathematiků” (Assotiation of Czech Mathematicians) started to issue “Časopis pro pěstování mathematiky a fysiky” (Journal for cultivating mathematics and fhysics). Except articles devoting mathematics and physics and the educational problems of this subjects, this journal featured specialized tasks solved by the students. The succesful ones were aworded mathematical books. Thousands of students took part in the solving of the above mentioned tasks during mote than fifty years existence of the journal. Some of them became well-known mathematicians (Sucharda, Lerch, Pleskot, Petr).
The tasks were created mostly by the secondary school teachers (Strnad, Sobička, Kostěnec, Pokorný), same of solvers became author of the tasks (Sucharda, Lerch, Pleskot) .
This event enhanced a lot the students' knowledge of Mathematics and it played almost as significant role as today's competition called “The Olympics in Mathematics”.
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***Theme* 6**

ERASMUS HABERMEL'S GEOMETRICAL SQUARE

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The Louvre Museum in Paris owns an excellent example of a geometrical square, created at the end of the 16^{th} century in Prague by Erasmus Habermel.
Habermel (ca 1550 – 1606) was named *astronomische und geometrische Instrumentenmacher* to Emperor Rudolf II's court. He was a contemporary of Kepler and Bürgi, and probably knew them well. His achievements in constructing scientific instruments are well known.
The geometrical square, dating back to the « middle ages », is based on a part of the back of the astrolabe and it consists of a mobile ruler and line or curved graduations. The use of it was well known, a variety of books about it could be found from the 15^{th} to the 18^{th} century.
The aim of the workshop is to make geometrical sense of this brass work of art, and to understand the way mathematics allowed people to measure distant lines, especially inaccessible ones, in their everyday lives.
To be studied in the workshop : original texts in French, Italian and German, translated into English : Jakob Koebel, Cosimo Bartoli, Oronce Fine, etc.
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***Theme* 6**

BERNARD BOLZANO AND THE MAKING OF MEANING IN MATHEMATICS

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Children enjoy playing with toys when they can give meanings to their interactions with the toys. It is similar with early encounters with language, number and shape. Although meanings begin as personal and subjective it is soon discovered that they may be shared and this process itself stimulates the growth of meanings. Learning is most successful when the student makes the (unending) journey from personal to public meanings.
Students who think they are ‘hopeless' or ‘bad' at mathematics have often not had the experiences needed to allow them even to make personal meanings of mathematical objects or ideas. I shall suggest that such personal meanings are never ‘received', but can only be ‘made' by the students themselves in response to appropriate experience.
Meanings begin as personal within individuals but become shared in communities. They therefore always arise within historical and cultural contexts.
At the beginning of the 19C there were several mathematical expressions, or concepts, for which there were no clear, accepted, public meanings. Examples are infinite series, the infinitesimal, terms with an irrational exponent and the derivative of a function. Of course, this situation was compatible with experienced mathematicians developing numerous successful theories and new insights involving these concepts. But those mathematicians were also like students learning – in their pursuit and negotiation of public meanings for what began as private interpretations.
An inspiring example of an educator and mathematician who explicitly struggled with the clarification of the meanings of new concepts is Bernard Bolzano (Prague, 1781 – 1848). This talk will address these issues in the context of Bolzano's little known early work on the binomial theorem where all four of the concepts mentioned above are explicitly addressed and substantial progress is made in their use and definition. The original German text of *Der binomische Lehrsatz* …., (1816) is currently difficult to obtain, but there is an English translation in my edition of, *The Mathematical Works of Bernard Bolzano* (OUP, 2004, reprinted 2006).
The mathematics we and our students are learning is full of living concepts and principles which are stable, but not static. They have historical roots and future evolution. The example of Bolzano's work will be used to

demonstrate the inevitable and organic connection of mathematics with other disciplines such as history, philosophy and language.
The overall theme of the talk in relation to mathematics education is not so much about history as an influence on current mathematics but rather to emphasise that all meanings (influential or not) must begin in a personal meaning that is made by the individual in a response to experience – a response that is inseparable from culture and context.
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***Theme* 6**

NAZI RULE AND TEACHING OF MATHEMATICS IN THE THIRD REICH

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The Nazi seizure of power in Germany in 1933 led to consequences for the teaching of mathematics on several levels. At schools and universities racially discriminated and politically unwanted teachers were dismissed, many women were deprived of their right to teach. In the choice of the subjects of mathematics teaching, the Nazi rule implied militaristic as well as racist and eugenicist assignments at schools and discouraged internationally flourishing fields like abstract topology and algebra on the university level. Perhaps the most important consequence for a subject such as mathematics was the general intellectual atmosphere, the one-sided support for physical and military training and the emphasis on "character" instead of "intellect" within the Hitler youth and on the eve of the World War.
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***Theme* 6**

ALGEBRA AND GEOMETRY IN OUR COUNTRIES AFTER THE ERLANGEN AND THE MERANO PROGRAMME

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The lecture will be dedicated to the influence of the Erlangen and the Merano programme on the development of algebra and geometry in Czech countries.
The Erlangen programme plays an important role in the development of mathematics in the 19. century. It is the tittle for the famous lecture of Felix Klein (1849–1925) *A Comparative Review of Recent Researches in Geometry*, which was performed in October 1872 at the University of Erlangen on the occasion of his admission as a professor. In this lecture F. Klein expounded an importance of the term „group" for the classification of various geometries. Klein's basic idea is that each geometry can be characterized by a group of transformations which preserve elementary properties of the given geometry.
At the end of his career Felix Klein was interested in teaching mathematics at German schools as well. He had been trying about its modernization and he made efforts for incorporation of the latest knowledge of mathematical science to classes at secondary schools and universities. From his direct initiative in 1905 in Merano a programme of restructuring of subject matter at secondary schools was formulated. This programme called for incorporation of the theme of groups of geometric transformations to the subject matter at grammar schools.
In this lecture basic data about Felix Klein will be mentioned and fundamental ideas about the Erlangen and the Merano programme will be presented. We will show afterwards how these ideas are reflected in Czech mathematics textbooks.
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***Theme* 6**

PRAGUE ET L'INFINI

Leonardo Venegas
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Après l'élan qui lui a été donné en terres grecques, il y a eu peu de villes où les problèmes concernant l'infini ont trouvé un accueil aussi sérieux et fécond qu'à Prague. C'est là où, sous la protection de l'empereur Rodolphe II, Giordano Bruno a rédigé ses *Articuli adversus mathematicos*, dans une atmosphère de recherche libre, unique à l'époque, ce qui lui a permis de pousser sa thèse la plus hardie, *De l'infinito universo e mondi*, publiée quatre ans avant, portant sur l'idée que l'univers non seulement est formé d'un espace infini changeant à perpétuité, mais qu'il contient aussi une infinité des mondes qui développent leur vie de manière simultanée. Allant contre la prescription d'Aristote, adoptée au sein de l'Église par Thomas d'Aquin et suivie dans toutes les universités d'alors, d'après laquelle les seuls processus effectivement exécutables ici bas sont d'ordre fini, il va de soi qu'une thèse pareille, avait du mal à trouver des partisans à Oxford, à Paris ou à Wittenberg, où la plupart des professeurs s'opposaient même à la théorie héliocentrique. En revanche, il est curieux de savoir qu'un autre hôte de l'empereur, le Rabbi Löw, talmudiste, mathématicien et philosophe néoplatonicien, a créé, rue des

Alchimistes, à Prague, à partir de l'argile rouge et des permutations kabbalistiques, le Golem, non pas la légende (ce qui serait la tâche des générations dépourvues des connaissances de l'infini leur permettant de franchir les limites imposées par la nature, donc de faire concurrence à la divinité), mais le vrai géant destiné à protéger la communauté juive.
Deux siècles après, le mathématicien et théologien Bernhard Bolzano, professeur de philosophie de la religion à l'université de Prague, nourri des œuvres de Cauchy et de Lagrange, se sentira persuadé que les contradictions auxquelles amène la logique de l'infini proviennent du fait d'appliquer à ce concept une pensée finitiste. Il franchira alors le seuil qui interdisait de décerner à l'infini le même statut des nombres finis, c'est-à-dire il le traitera comme une réalité actuelle, au sens d'Aristote, et non seulement comme un processus sans terme. À la fin de sa vie il construira un ensemble de paradoxes de l'infini, qui, bien interprétés, ne seront plus des contradictions mais plutôt des propriétés d'un concept maintenant devenu légitime. On sait qu'il faudra attendre Cantor pour trouver une vision semblable sur ce point, quoique l'un et l'autre ont vécu presque sans avoir un interlocuteur avec qui partager leurs travaux. Le dernier solitaire de cette liste est Kafka, dont l'œuvre, forgée avec la lucidité des insomniaques, nous donne l'impression d'être la lutte d'un homme, muni seulement de sa pauvre existence, pour atteindre des réponses qui lui échappent au fur et à mesure qu'il s'avance vers elles, comme dans le paradoxe de Zénon.
Dans ce travail nous essayerons de montrer que les approches de la métaphysique, de la légende, de la logique et de la littérature enrichissent un sujet où il y a d'autres coïncidences en plus de la belle ville qui a abrité son étude.

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Abdounur Oscar João	An exhibition as a tool to approach didactical and historical aspects of the relationship between mathematics and music	2	English	Brazil
Auvinet Jérôme	C. A. Laisant viewed through his book "La mathumatique, Philosophie, Enseignement"	5	English	France
Bertato Fábio Maia	A Classroom Experience: Using a Computer Role-Playing Game to teach Mathematics	2	English	Brazil
Bjarnadóttir Kristín	The Number Concept and the Role of Zero in European Arithmetic Textbooks from the Twelfth to the Nineteenth Century	5	English	Iceland
Cihlar Jiri, Eisenmann, Petr, Kratka Magdalena, Vopenka Petr	A coherence of ontogeny and phylogeny within the context of a problem of a point distribution on a segment	1	English	Czech Republic
Čmejrková Kamila	Solving logical problems and Commutative skills for the groups of pupils	3	English	Czech Republic
Costa Cecília	Introducing a historical dimension into teaching: A Portuguese example – J. Vicente Gonçalves	5	English	Portugal
Costa Cecília, Catarino Paula, da Silva Nascimento Maria Manuel	Could mathematics transform my land in the capital of universe?	4	English	Portugal
da Graça Alves Maria	The first Portuguese Mathematical Journal: _The "Jornal das Sciencias Mathematicas e Astronomicas" de Francisco Gomes Teixeira	4	English	Portugal
Gomes Alexandra, Ralha Elfrida	Mathematical training and primary school teachers: where are we coming from and where are we going to, in Portugal?	5	English	Portugal
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Hitchcock Gavin A.	Analysis with the Help of its History	2	English	Zimbabwe
Isaias Miranda	Linear movement of objects: Differences between Aristotle's and high school students' analysis	1	English	Mexico
Jankvist Uffe Thomas	A teaching module on the early history of error correcting codes	2	English	Denmark
Marikyan Gohar	Anania Shirakatsi's 6th Century Methodology of Teaching Arithmetics Across the Centuries and Divers Cultures	4	English	USA
Mercurio Anna Maria, Palladino Nicla	On the resolution of algebraic equations	1	English	Italy
Ozkan E. Mehmet, Unal Hasan	Influence of mathematicians in history on pre-service mathematics teachers' beliefs about the nature of mathematics	3	English	Turkey
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Saidkarimov Utkur	Exhibition of didactical material, relevant to the ESU-5 main themes	1&4	English	Uzbekistan
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Spyrou Panayotis, Lappas Dionyssios, Keisoglou Stefanos	Embodied Mathematics in a Historical Perspectives	1	English	Greece
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Short Presentations
ABSTRACTS

(ordered by theme 1, 2, 3, 4, 5, 6)

Theme 1

Theme 1

A COHERENCE OF ONTOGENY AND PHYLOGENY WITHIN THE CONTEXT OF A PROBLEM OF A POINT DISTRIBUTION ON A SEGMENT

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The paper deals with an analysis of the possible approaches to a solution of the following problem: A square ABCD is given. Find an X point on its side BC, so that the triangle ABX is of the minimal area. Our aim is to find a coherence of the solution approaches to a problem in terms of phylogeny and ontogeny. The phylogenetic approach is characterised by a hypothetical solution as to how it could be solved e.g. by Euclid, Domecritos, Kepler, set or school mathematics. The ontogenetic approach is characterised by typical reactions of a contemporary individual at a particular level of development as they were recorded during the experiments.

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***Theme* 1**

LINEAR MOVEMENT OF OBJECTS: DIFFERENCES BETWEEN ARISTOTLE'S AND HIGH SCHOOL STUDENTS' ANALYSIS

Isaias Miranda
Laurentian University, Canada; Cinvestav-IPN, México

The main purpose of studying the relationship between Aristotle's and high school students' conception of motion has been to argue that the latter explain the movement of objects in a similar way to the former. However, in regard to linear movement the argument seems unsupported. Students and Aristotle completely differ in their ways of both representing linear movement and conceiving motion. In this research I examine Aristotle's concept of motion in order to discover differences between his analysis and students' description of linear movement. It is argued that in order to create a coherent philosophical system, Aristotle needed to postulate linear motion occurs in a line that is continuous but does not consist of points. This assumption is completely different from high school students' conceptualization of linear motion; for they are told that it occurs in a straight line that is continuous and composed of points. By this analysis, it may be possible to have a better understanding of students' learning of motion.

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***Theme* 1**

ON THE RESOLUTION OF ALGEBRAIC EQUATIONS

Anna Maria Mercurio, Nicla Palladino
Università degli Studi di Salerno, Italy

In Italian high school, first and second-degree algebraic equations and procedures to solve them by radicals are proposed. Seldom there is mention about the solutions of cubic or quartic equations (solvable by Cardano and Ferrari formulas) and only some particular cases are introduced to students. Thus there is not enough emphasis on the research for the solutions of higher than four-degree general algebraic equations. As a consequence, there is the erroneous belief that these kinds of equations are always solvable by radicals and so let unknown almost two centuries of discussion about this subject.

Recently, we worked about the research for the solutions of quintic equation, principally focused on the crucial years between 1850 and 1860. During these years, Enrico Betti developed Galois' ideas more organically, and Hermite, Kronecker and Brioschi published their fundamental works on the research for the solutions of quintic algebraic equations by the support of Galois' theory and elliptical functions.

Through our historical researches about these works and correspondences among Betti, Brioschi, Hermite and Kronecker, we remarked that they got into difficulties to obtain concrete solutions.

So, this fact inspired us to propose historical considerations on these questions that could be illustrated to high school students so that they don't fall in the erroneous belief that all the general algebraic equations of whatever degree are always solvable using radicals.

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***Theme* 1**

ÉVARISTE GALOIS' ALGEBRAIC THEORY, EPISTEMOLOGICAL REFLECTIONS AND EDUCATIONAL ELEMENTS

Raffaele Pisano Nicla Palladino
University of Rome "La Sapienza" – University of Salerno

Up until the 17th century, attempts to produce arguments on infinitely great and infinitely small quantities were numerous and multifarious with the most vital contributions coming exclusively from Newton (1642-1727) and Leibniz' (1646-1716) theories. Nevertheless the logical-mathematical and physical meaning of infinitesimal objects was still not specified, in an area which itself remained undefined conceptually. As a consequence, this way of conceiving the mathematical sciences and for interpreting physical phenomena (e.g., thermodynamics) produced, in the 19th century, well-known speculations on metaphysical objects ($\neq 0?\ \rightarrow 0?\ 0?$). In the tension-filled atmosphere of the era, Évariste Galois (1811-1832) played an important role proposing reasoning, as well as, a revolutionary thesis both for his predecessors and contemporaries. Recent historical and educational studies have also confirmed that researchers at the beginning of the 19th century had difficulty understanding his unusual conceptual framework; and that his thesis seemed more consistent with mathematicians of the mid-20th century, than *rigorous calculus* of his era.
Here, we analyze the historical development of the foundations of the theory expressed by the reasoning in his *Écrits et mémoirs mathématiques*. Our investigation takes us through two categories of historical interpretation: the order of ideas as an element for understanding the evolution of scientific thought on one hand, and the use of logic as an element for scanning and controlling the organization of the theory on the other. The use of categories is valid since the historical exploration of the foundations will not be analyzed using the traditional approach. Obviously the content of this work could appear potentially factious, since it cannot be assumed to be the only possible perspective**.**

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***Theme* 1**

TEXTS, TEXTUALITY AND MATHEMATICAL SYMBOLS

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The invention of printing in the Renaissance brought forward new forms of knowledge representation that led to changes in cognition (McLuhan, 1962). What constituted mathematics and mathematical activity had to be put into suitably compacted signs. It is only in this context that the emergence of algebraic symbolism can be properly understood. The whole arsenal of resources used in what has been called *oral algebra* (Radford, 2006) — vocal inflections and rhythms, gestures, artefact-mediated and bodily actions— was substantially reduced, if not replaced, by the written text. As a result, the contextual dialectics of face-to-face interaction at the base of mathematical activity changed.

In contemporary mathematics classrooms, we observe that, as soon as the activity is not focused on written symbols only, the students resort to an intertwining variety of semiotic means (language, gestures, diagrams, symbols, actions …) that evokes the context of oral algebra.

We submit here a new idea of mathematical text to account for the richness of mathematical activity as it may have developed in the history of mathematics. A mathematical text would have a spatio-temporal "textuality": it would not only include written symbols but also gestures, and other types of signs through which mathematical thinking occurs. Through a historical reconstruction of Nichomachus of Geresa's teachings we provide an example of "enlarged text" and endeavour to show the textuality that Nichomachus' text may have encompassed. Some contemporary examples are also shown through classroom video clips.

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***Theme* 1**

MATHEMATICS, PHYSICS AND ART
(FUNCTIONS, SIMPLE PENDULUM, ART)

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One of the major subjects of the mathematics is functions. That subject can be seen at every place of daily life. To teach function subject, teachers give examples from daily life and as a science from physics. Physics formulas and rules are good examples of the functions. One of them is the simple pendulum experiment. Simple pendulum's history was an good example of the explain the origins of subjects. The past and today connection also made by giving today's examples of simple pendulum. During the making experiment, students recognized the relation between mathematics and physics and also they fell that they can touch functions so mathematics. After the experiment, students construct a formula depending on the data sheets. That formula is an example of the functions. Students checked two main requirements of being functions using the graphs of a simple pendulum. The graphs were drawn by using experiment data sheets. One simple pendulum was developed functions, what about do the 2 simple pendulum come together? The two simple pendulum comes together to form a harmonograph. Harmonograph showed the students how mathematical art constituted. Students see the connection of mathematics with the physics and so art. They took the notice of the connection between the subjects.

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***Theme* 1**

EMBODIED MATHEMATICS IN A HISTORICAL PERSPECTIVE

Panayotis Spyrou, Dionyssios Lappas, Stefanos Keisoglou
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The theory of embodied mathematics was proposed by Lakoff & Núñez (2000). We suggest that the "embodied mathematics", should be additionally studied in a historical direction. The main idea supported here is that the embodiment arises when the mind tries to make the subjective sense an objective one of central perceptual apparatus and encapsulated it in concepts and representations. We assert that the main mechanisms of our spatial perception are the ability to perceive the gravity, the similarity of shapes and the depth of the three-dimensional space. In Lappas & Spyrou (2006) have shown how the main theoretical instruments of Euclidean Geometry are produced around *verticality, horizontality* and *geometric similarit*y. According to this view Pythagorean and Thales' Theorem are the conceptual representation of our sense capabilities. In this respect is analyzed a classical results from Euclid's Optics (statement 4). This is the first effort to study the relation between an angle and the height to the one opposite side: "Let us consider equal segments which lie on the same straight line. The ones viewed at greater distance appear smaller." Next we connect this fact with a treatment used by Dürer that was a presage for the concept of tangent.

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***Theme* 1**

GEOMETRICAL APPROACH OF DIFFERENTIAL EQUATIONS: FROM HISTORY TO MATHEMATICS EDUCATION

Dominique Tournès
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For ten years, I have conceived and I tried out activities of geometrical approach of the differential equations, at the same time in classes of secondary schools and in teachers' training. I am inspired for that by the methods of construction of curves imagined by the pioneers of calculus (Newton, Leibniz, Euler, Riccati…), as well as graphic methods of calculation practised by the engineers of the 19th century: construction by segments of tangents, by arcs of osculatory circles, by tractional movement, etc. The activities, which result from it, appeal only to elementary geometrical knowledge and can naturally be enriched by the use of modern dynamic geometry software. They allow the pupils to acquire a simple and natural geometrical vision of the concept of differential equation, in conformity with the historical process of its development and likely to prepare effectively the later analytical study. My talk will offer a short panorama of the possible activities and the experiences already carried out in this field.

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Theme 2

***Theme* 2**

AN EXHIBITION AS A TOOL TO APPROACH DIDACTICAL AND HISTORICAL ASPECTS OF THE RELATIONSHIP BETWEEN MATHEMATICS AND MUSIC

Oscar João Abdounur

Instituto de Matemática da Universidade de São Paulo

In this presentation, we propose the use of an exhibition to approach historical and didactical aspects of the relationship between mathematics and music. In establishing a context for teachers to experience activities of culture and extension to their curricular activities, one values the history of mathematics particularly concerning its relationships with music, making accessible the historical context in which such relationships emerged. One proposes the experience of situations historically contextualized involving simultaneously mathematical, physical and musical concepts, be it directly, be it by means of analogical reproductions that intend to unchain the interest and reflection for its study.

Under a historical-didactical perspective, this presentation proposes the exhibition by means of eight parts that intends to transmit central ideas of the relationship between mathematics and music: 1) Motivation for the understanding of the Harmonic Series; 2) The experiment of the monochord: ratios x musical intervals in the mathematical systematization of the scale; 3) Renaissance: the relationship mathematics-music as experimental science; 4) Mathematical systematization of scales and temperament: ratios, irrational numbers and logarithms; 5) Harmonic Series/Fourier Series; 6) Consonance and dissonance: from arithmetical symbolism to a physical conception; 7) The sound of the planets; 8) From speculative mathematics to empirical mathematics: a scientific revolution in music.

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***Theme* 2**

A CLASSROOM EXPERIENCE: USING A COMPUTER ROLE-PLAYING GAME TO TEACH MATHEMATICS

Fábio Maia Bertato

Centre for Logic Epistemology and History of Science (CLE/Unicamp – Brazil) & Colégio Puríssimo Coração de Maria (Rio Claro - Brazil)

Our objective is to present an experience using the creation of a computer role-playing game as motivation to study mathematics and its history. A group of high school students of the *Colégio Puríssimo Coração de Maria* (Rio Claro - Brazil) was oriented in the creation of a game. This project was called *Gemina: RPG and Mathematics* and it was divided in three phases: (1) the creation of a story about *Gemina*, a universe governed by mathematical principles; (2) the development of a computer game with the software RPG Maker; (3) the organization of an event to introduce *Gemina* to the others students. The characters of the game were based on historical figures as Euclid, Thales of Mileto, Pythagoras, Leonardo Fibonacci, Pacioli, Cardan, Tartaglia, etc. Besides a historical research to creation of the story, characters, worlds and dialogues, a study on mathematical and logical contents of the adventure was made.

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***Theme* 2**

ANALYSIS WITH THE HELP OF ITS HISTORY

Gavin A. Hitchcock

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I have for some years taught a second year and following third year University course on Analysis, and am close to completing a textbook with the title of this talk. I will describe some teaching experiences and student responses to both the classroom interaction and the text, which I have issued in draft form.

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***Theme* 2**

A TEACHING MODULE ON THE EARLY HISTORY OF ERROR CORRECTING CODES

Uffe Thomas Jankvist

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Regarding the use of history in mathematics education enough has, according to Siu (1998), already been said on the propagandistic level. What is lacking is investigations on the effectiveness of such use. My Ph.D. research includes two investigations on upper secondary level, one of which concerns an evaluation of a teaching module on the early history of error correcting codes (elements of Shannon's, Hamming's and Golay's work). The implementation and evaluation of this 15-lesson module is to take place in April 2007.
The subject of the module is extra-curricular and serves as one way of realising the now required element of the history of mathematics in the Danish upper secondary mathematics programme. The purpose is to let history serve as the *aim* (or goal) instead of, what is often seen, an *aid* (or tool) for learning mathematics better. Some of the aims include showing the students that mathematics is still being developed, how a mathematical discipline may be born due to practical needs, and to show them that mathematics is in fact used in our everyday lives - although it may be hidden. Since the module includes both history and application aspects one of my problems on the evaluation part is foreseen to be distinguishing between the effectiveness of the two.
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***Theme* 2**

MATHEMATICS THEATER - MATHEMATICIANS ON STAGE

Funda Gonulates
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There is a common complain and fear about mathematics. Showing the human side of mathematics to students, how mathematical ideas are evolved, the struggles in history to create mathematical facts can be integrated in teaching and learning cycle. Mathematics history can be integrated into the mathematics courses via the introduction of famous mathematicians. My students met with mathematicians via Mathematics Theater. They had made searches about life of different mathematicians, they learnt how they contributed mathematics and they wrote scenarios about those mathematicians and as a final step they performed the lives of those mathematicians as a theater on the stage. Evariste Galois, Sophie Germain, Pythagoras and Archimedes were the mathematicians included in the math theater. 5th and 6th grade students were the participants of this activity. They took responsibility in every part. The play they were performing was their own play in every aspect. By this way they learnt about four different mathematicians; they experienced how a mathematician could live, make mathematics; how mathematics could inspire people and so on. As a teacher I know that my students challenged the myth that mathematics is not a perfectly finished body of knowledge or can not be changed or mathematics is the product of somebody out of this universe. They were the mathematicians on the stage and they told this to their friends also by performing a theater.
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***Theme* 2**

RAMANUJAN REVISITED

Jim Tattersall
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Among mathematicians, in particular number theorists, Ramanujan’s mathematical accomplishments during his short life are legend. However, few if any of his results are included in the undergraduate mathematics curriculum. His work on highly composite numbers and magic squares is appropriate for courses for liberal arts majors. We give a brief history of the development of the two topics before focusing on his innovative constructions of magic squares done in his early school days. The material present is interesting, intriguing, and apropos for high school students as well as for undergraduates. It can also be used to create engaging research projects for undergraduate mathematics majors.
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***Theme* 2**

A TEACHING EXPERIENCE WITH A HIGH-LEVEL GROUP OF STUDENTS ABOUT THE HISTORY OF MATHEMATICAL METHODS IN APPROACHING THE CONCEPTS OF AREA AND VOLUME

Luciana Zuccheri
Dipartimento di Matematica e Informatica, University of Trieste, Trieste, Italy
Paola Gallopin
Liceo Scientifico “Galilei Galilei”, Trieste, Italy

We present a teaching experience, carried out already two times in the last two years in a Scientific Italian Lyceum, with high level groups of students (16-19 aged). The subject proposed to the students focuses on the history of integral calculus, in particular on the exhaustion method and on the indivisibles method, in approaching the concepts of area and volume. The experience was prepared in co-operation by an university professor and a group of secondary school teachers who participated in the Trieste Unit of the national educational project “Progetto lauree scientifiche” (Project for the scientific degrees, http://www.laureescientifiche.units.it/) for mathematics, which aims to prompt the best secondary school students to mathematics studies. The students involved in the experience worked using co-operative methodologies, analysing some texts from the Euclid's Elements and from Archimedes' works to learn the bases of the exhaustion method, and from the works by Galileo, Cavalieri and Torricelli for the indivisibles method. At the end of both experiences we submitted a questionnaire to the students and recorded a discussion with them about the work carried out. We illustrate here the details of these teaching experiences and evaluate them analysing the collected data.

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Theme 3

***Theme* 3**

SOLVING LOGICAL PROBLEMS AND COMMUTATIVE SKILLS FOR THE GROUPS OF PUPILS

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The goal of the thesis is to study approaches used for solving non-standard logical problems by the groups of pupils, to retrieve and analyze common patterns of communication, and to study the dependencies between the communicational skills in the group and the achieved success. The problem has also its historical background: The non-standard problems focused on the logical reasoning appear in the entire phylogenesis of the mathematics, as well as the problems of communication and sharing information on the reasoning process can be found in works of well known mathematicians over the course of the evolution of mathematics and logic. The method of analyzing the data, especially the atomic analysis is rooted in the techniques of the structural linguistics – an approach to the text structure introduced at the beginning of the 20th century in linguistics and semiotics (Ferdinand de Saussure, The Prague Linguistic Circle, etc.), and in many other fields of research.
The key part of my work is an evaluation of results. Over the course of the experiment, small groups of either two or four pupils of elementary school, aged 13-15 years solved logical problems. Their task was to solve a set of problems together, to intercommunicate and to explain their approaches to each other, so that every member of the group understood the reasoning sequence that had led to the solution.
With respect to the fact that logic is not a subject taught at an elementary school, we had supposed that the pupils would use their intuition and common sense based on their cognitive competence when solving the problems. We created a corpus of recordings and transcriptions of the pupils' dialogues. We analyzed the data using the speech act theory, conversational analysis, and the argumentation theory. For the analysis of the solving process, we used the method of the atomic analysis.
The conclusions based on our observations show that the success of solving the problems depends on the language and cognitive competence of the members of the groups as well as on the communication behavior in the groups. The abilities of analyzing the text of the problem, comprehension and grasping all objects and relations among them appear as the most useful ones. The analysis of the text is complicated by the expressions that are not usual in common language, or that have a different meaning or strong connotations, as well as by the complexity of the analyzed text (the length of the text and the number of subjects appearing in the task).
Our analysis of the speech acts shows a relation between utterances functioning as explanation, reasoning, argumentation, and the success rate of the group. The sequential analysis also shows that certain speech sequences are significant for more or less successful groups, respectively. Generally, it is possible to conclude that the maximization of shared information is a good prerequisite of a successful group.

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***Theme* 3**

INFLUENCE OF MATHEMATICIANS IN HISTORY ON PRESERVICE MATHEMATICS TEACHERS' BELIEFS ABOUT THE NATURE OF MATHEMATICS

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The purpose of this study was to investigate preservice secondary mathematics teachers' beliefs about the nature of mathematics and connections with their favorite mathematicians in history. More specifically the study examined the connection between the Preservice mathematics teachers' favorite mathematicians (characteristics, background, pure& applied, etc.) and their conception of what mathematics is.
Teacher beliefs and practices as a research domain gained much attention over the last two decades (Unal & Jakubowski, 2005). In previous study Jakubowski and Unal (2003) have demonstrated that beliefs about nature of mathematics influence knowledge acquisition and interpretation, task definition and selection and interpretation of course content. They found that formal education program and faculty had an effect on shaping teacher's beliefs and classroom actions. According to Tobin and Jakubowski (1990), the view a teacher holds of mathematics and science influence classroom interactions and teaching goals. In general, teacher beliefs can have a strong influence on teachers' approach to teaching mathematics.
The study is qualitative in nature. The framework used to analyze data for the study is the Rokeach's (1968) belief system model. Data, gathered over the two spring semesters, included videotaped interviews, document

analysis, drawings (picture of mathematics). Researchers have analyzed the data jointly. The findings of the study will be discussed in detail.

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***Theme* 3**

SOME QUESTIONS REGARDING THE HISTORICAL ROLE OF CONSTRUCTIVISM IN MATHEMATICS EDUCATION REFORM

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In this article, I will raise some questions that could serve as an agenda for a future historian looking back at the role played by constructivism in U.S. math education reform during the last quarter of the twentieth century. I explain each question in detail and also explain why I think the question is significant and why I do not think its answer is obvious.

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***Theme* 3**

THE QUESTION OF CHANGING MATHEMATICS SECONDARY SCHOOL CURRICULA IN VENEZIA GIULIA AFTER THE FIRST WORLD WAR (1918-1923)

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We present a research focused on the question of changing mathematics secondary school curricula in Venezia Giulia, Northern Italy, after the First World War, in the period of transition from the regulations of the Habsburg Empire to the ones under the Reign of Italy (from 1918 to 1923).

This question was at that moment of great topical interest to mathematics education in Italy, where the voice of a reformist current was strong and supported a less theoretical and more practical teaching, in the wider context of an European reform movement.

Our research deals in particular with the work of the "Mathesis" Society, Section of Trieste, and is supported by publications of those days, school yearbooks, documents of the State Archives of Trieste and of the Municipal District of Trieste and the archives of the "Mathesis" Triestine Section (*Fondo Mathesis*) kept at the University of Trieste.

The research shows that, in spite of their strong Italian feelings, the mathematics teachers of Trieste, trained at the Austrian universities, where they learned the teaching methods proposed by Felix Klein, did not accept passively the change of the mathematics secondary school curricula, until the Gentile Reform (1923) obliged them to.

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Theme 4

Theme 4

COULD MATHEMATICS TRANSFORMS MY LAND IN THE CAPITAL OF UNIVERSE?*

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The northeast of Portugal, known as Trás-os-Montes e Alto Douro, is a region with characteristics well marked, which promoted a secular kind of culture, quite different from the remaining country.

This region is well known for the production of Porto wine, refined olive oil, gastronomic specialities, pottery, etc. Therefore there are several human activities involved with this sort of productions. Many of these activities are at risk of extinction.
Our main goal is to apply new methodologies of learning mathematics in classroom focused in the natural environment and cultural context of the students. We are developing a project with 6 schools in this region, involving almost 500 students of ages between 11 to 15 years old. With the help of their teachers, and the project team, different groups of students choose a traditional activity and identify some of the mathematical processes involved in it. They do visits to observe the locals and to interview the artisans. After what they analyse and interpreted the data collected and produce some didactical materials to show and disseminate their conclusions.
We also hope to contribute to the promotion of secular activities of this region, which are almost extinct, near the young students' native population.

* Project "Ciência Viva VI nº 771"

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Theme 4

THE FIRST PORTUGUESE MATHEMATICAL JOURNAL: THE "JORNAL DAS SCIENCIAS MATHEMATICAS E ASTRONOMICAS" DE FRANCISCO GOMES TEIXEIRA

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Francisco Gomes Teixeira (1851-1933) is a great figure of Portuguese mathematics. In 1876, he obtained his PhD in the Universidade de Coimbra.
Later, he would publish hundreds of papers in several and reputed mathematical journals in the area of Infinitesimal Analysis. He would also write important text books, his famous *Traité des Courbes Spéciales Remarquables Planes et Gauches* and also on the history of mathematics.
But the next year of his PhD, in 1877, still very young, he was able of evaluating the situation of total insulation of the Portuguese mathematical community. So he took on his shoulders the enormous task of the foundation of the first Portuguese mathematical journal, that he called "Jornal das Sciencias Mathematicas e Astronomicas", later on also known by Teixeira's Journal, entirely supported by the Universidade de Coimbra.
This journal was a landmark, not only in the international spreading of Portuguese mathematics and Portuguese mathematicians in the mathematical community, but also in the spreading of foreign mathematicians and their works in Portugal. It deserved a place in the list of celebrated mathematical journals that is fair to be made known.

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Theme 4

ANANIA SHIRAKATSI'S 6TH CENTURY METHODOLOGY OF TEACHING ARITHMETICS ACROSS THE CENTURIES AND DIVERS CULTURES

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In [1] Anania Shirakatsi describes methods he has used in teaching arithmetic to first grade children. This manuscript contains interesting detailed explanations. His methods has been used for centuries in Armenia. The net effect of his teachings has resulted in acquiring a high grade of arithmetic knowledge in children with a strong foundation for subsequent achievements in scientific research. Over my teaching years in the U.S.A. I have had the opportunity to observe and compare the American system of introducing arithmetic to school children with the methods used in Armenia. Anania Shirakatsi's teaching methodology can be as effective in the contemporary diverse classroom as it has been since the 6th century.

[1] Anania Shirakatsi, Tvabanutiun (Arithmetic), 6th Century

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Theme 4

ON THE PORTUGUESE MATHEMATICAL READINGS ABOUT THE GREGORIAN CALENDAR REFORM

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Several attempts and reforms had been made over the centuries to adjust theoretical/arithmetical cycles to astronomic events.

In 1582, Bull *Inter gravissimas* (by Pope Gregory XIII) replaced the Julian Calendar by the, so called, Gregorian Calendar. Once political/institutional decision was taken, it urged to convey the new rules of *computus* into people's minds and several mathematical works were published in the following decades. The Jesuit Christophoro Clavius became the main defender of that reform, which was formerly based on a Luigi Lilio's project.
We will focus our presentation on both the Portuguese immediate acceptance of the new (Gregorian) Calendar and on its evocative mathematical works, namely *Chronografia ou Reportorio dos tempos* (Avelar, 1585), *Chronografia: reportorio dos tempos* (Figueiredo, 1603) and *Thesouro de prudentes* (Sequeira, 1612). We aim at underlining the perspective of practical mathematics embedded over a social permanent need: to measure time.
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Theme 4

SUR L'ÉTUDE DES CRÉPUSCULES POUR PEDRO NUNES

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Dans cette présentation, on fait quelques considérations, sur ce qui concerne la dépression du soleil, au commencement du crépuscule matutin et à la fin du vespertin. Pedro Nunes (1502-1578) montra que cette dépression-là n´était pas constante, comme l´on pensait des l´Antiquité, mais variable, chaqu´endroit, de jour en jour.

Theme 5

Theme 5

C. A. LAISANT VIEWED THROUGH HIS BOOK "LA MATHEMATIQUE, PHILOSOPHIE, ENSEIGNEMENT"

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En 1898, Charles-Ange Laisant publie *La Mathématique, Philosophie - Enseignement*. L'ancien député de Nantes se consacre alors à sa carrière d'enseignant notamment au sein de l'Ecole Polytechnique et l'ouvrage marque le début d'une nouvelle période où domine une réflexion plus large d'un mathématicien accompli sur son domaine (avec, par exemple, la création de la revue *L'Enseignement Mathématique* en 1899). Soulignant les transformations qui sont intervenues au cours du XIXème siècle dans l'industrie, Laisant conclut à une nécessaire modernisation de la vision et de l'enseignement des mathématiques. C'est l'unité de « *La Mathématique* » qui est prônée ici, tout autant que l'origine expérimentale des définitions géométriques ou arithmétiques. C.-A. Laisant poursuit ici auprès des étudiants, des enseignants et des ingénieurs son œuvre de diffusion entrepris notamment au sein de l'AFAS. Et c'est bien ses idées sociales qui guident le mathématicien dans la rédaction de *La Mathématique…* quelques années avant les transformations de 1902.
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Theme 5

THE NUMBER CONCEPT AND THE ROLE OF ZERO IN EUROPEAN ARITHMETIC TEXTBOOKS FROM THE TWELFTH TO THE NINETEENTH CENTURY

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In *Algorismus*, a short treatise in old Norse manuscripts from the fourteenth century, says that one is not a number but the origin of all numbers. Furthermore it says that *siffra*, the zero, is a figure that means nothing for itself, but that it marks space and adds meaning to other figures. *Algorismus* explains Hindu-Arabic arithmetic, but the number concept presented there is of Greek origin. At a closer look, similar ideas about the number concept and the role of zero can be traced through a number of European textbooks from the twelfth to the nineteeth century. Among them are Dixit Algorismi, a Latin translation of Alkwarizmi's arithmetic textbook, some Danish and German textbooks from the seventeenth and eighteenth century and several written and printed textbooks in Icelandic from the eighteenth and nineteenth century. The origin and evolution of these ideas will be discussed.
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Theme 5

INTRODUCING A HISTORICAL DIMENSION INTO TEACHING:
A PORTUGUESE EXAMPLE – J. VICENTE GONÇALVES.

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It could be useful to understand how a significant Portuguese mathematician and professor, of the first half of the twentieth century, introduced a historical dimension into the process of teaching and learning mathematics in his courses. Notice that this occurred before “History of Mathematics” becoming a formal discipline in Portuguese curricula.
I am referring to J. Vicente Gonçalves (1896-1985). He taught for almost 25 years in each of the biggest Portuguese universities: Coimbra University (1917-1942) and Lisbon University (1942-1967). In this last period he also taught at an important high school of economics (1947-1960). He wrote several mathematic textbooks for the secondary level and the university level, which have been (and some still are) used by many students.
Analysing J. Vicente Gonçalves way of teaching and his textbooks, I verified that he included regularly the historical dimension in his courses and in different ways:
-as brief historical notes: about mathematicians, about the introduction of notations and his authors, about the creation and evolution of mathematical symbols**,** etc.;
-as introduction of new concepts;
-as a pedagogical tool;
-as a content: referring to some historical facts;
-as resource of exercises (exercises based or adapted from old mathematical texts).
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***Theme* 5**

MATHEMATICAL TRAINING AND PRIMARY SCHOOL TEACHERS: WHERE ARE WE COMING FROM AND WHERE ARE WE GOING TO, IN PORTUGAL?

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The fact that anyone who has to teach mathematics should have a good mathematical training is unquestionable but, in the case of primary school teachers, this training seems even more crucial since they are responsible for the beginning of a desirable long period of mathematical learning and for the introduction of elementary concepts that will serve as foundations for the whole mathematical building. On the other hand, the idea of easiness associated to this mathematical content has been refuted by several researchers.
Nevertheless, the reality is that, at least in Portugal, the mathematical training of primary school teachers has been neglected for quite some time.
Recently, the Portuguese government, facing the serious problem of the persistent bad results in mathematics achieved by our pupils, decided to launch an in-service teacher training in mathematics for primary school teachers. We will reflect upon the present situation concerning the mathematical training of primary teachers supported by data collected on a research study which involved both university students (future primary school teachers) and primary school teachers with different initial mathematical background and we will explore their mathematical knowledge as well as their attitude towards their professional future.

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***Theme* 5**

EVIDENCE AND CULTURE, RIGOR AND PEDAGOGY: EUCLID AND ARNAULD

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Euclid's Elements begin with definitions that seem to be obvious and out of time. Nevertheless, don't they depend on a common culture of the ancient Greek world? In the 17th century, Arnauld wrote "New elements" which are based on a new order and a new way of thinking evidence. He says that, in this manner, the rigor of the demonstration and the easiness of understanding can go together.
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***Theme* 5**

THE EVOLUTION IN THE INTRODUCTION OF LEARNING STYLE IN THE TEACHING OF CALCULUS IN MEXICO

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In the last century several authors had been working about the learning style, Dewey, Kolb etc. In particular, this work follows the methodology of Bernice McCarthy, the 4MAT methodology of learning styles. This is a general methodology, but in the last years has been used for the teaching of mathematics.
In Mexico, the pioneer in the use of the 4MAT methodology was Tecnológico de Monterrey (ITESM), later the Insituto Politécnico Nacional (IPN) began to use for the teaching in Calculus. The present work show the evolution in the teaching of calculus since the introduction of the 4MAT methodology making a comparative study between the original course and the present course that include several activities around the four styles without favoring one of them. The history of this evolution in Mexico touches necessarily the IPN because this is the rector institution in the teaching of the engineering in Mexico.

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