

The International Commission on Mathematical Instruction
ICMI
Quadrennial Report by HPM
The International Study Group on the Relations between
History and Pedagogy of Mathematics
(HPM Study Group)

HPM Activities 2004-2008

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Over the past four years, since the last General Assembly of ICMI in Copenhagen during ICME 10 and the last Satellite Meeting of the HPM Group in Uppsala, Sweden, there have been several activities by this group, which either continue work that has been in progress, or constitute new ones based on initiatives by people active in this area. These include conferences and publications, as well as the improvement of the HPM Newsletter and the HPM websites, by making systematic use of the Internet. There has been an effort

- to regularly publish and continuously enrich the *HPM Newsletter*, as well as, constantly improve and update the *HPM websites*. Both the *Newsletter* and the *websites* have been developed as important tools complementary to each other, for making easier the contact among the members of the group and for increasing its visibility.
- to actively support magazines, or journals related to the *HPM* perspective and encourage, or motivate the publication of special issues of other journals devoted to themes that emphasize the historical dimension in Mathematics Education.
- to collaborate in the organization of local, or international activities (meetings, conferences, colloquia etc) that provided the opportunity to bring together people, who are interested in integrating history into Mathematics Education. Such activities stimulate new collaborations and foster further initiatives that bring closer, mathematicians, historians of Mathematics and mathematics educators, who are eager to contribute to the improvement of Mathematics Education around the world.

1. The structure of HPM

The *HPM* Advisory Board has been enlarged to have a sufficiently good geographic representation, as well as, a balanced representation of the three dimensions of *HPM* (*History, Pedagogy, Mathematics*). The Advisory Board has consisted of:

- Abraham Arcavi, Weizmann Institute of Science, Israel
- Evelyne Barbin, IREM-Centre François Viète, Université de Nantes, France
- Ricardo Cantoral, Departamento de Matematica Educativa, Centro de Investigacion y de Estudios Avanzados del IPN, Mexico
- Ubiratan d' Ambrosio, Pontificia Universidade, Catolica de Sao Paulo, Brazil (former chair 1984-88)
- Abdellah Elidrissi, Ecole Normal Supérieure, Morocco
- Florence Fasanelli, American Association for the Advancement of Science, USA (former chair 1988-92)
- Gail FitzSimons, Monash University, Australia
- Fulvia Furinghetti, Department of Mathematics, Università di Genova, Italy (former chair 2000-04)
- Wann-Sheng Horng, National Taiwan Normal University, Taiwan

- Masami Isoda, Graduate School of Comprehensive Human Science, University of Tsukuba, Japan
- Sten Kaijser, Department of Mathematics, University of Uppsala, Sweden
- Nikos Kastanis, Department of Mathematics, University of Thessaloniki, Greece (Newsletter co-editor)
- Victor Katz, Department of Mathematics, University of the District of Columbia, USA
- Manfred Kronfellner, Institut für Algebra & Computermathematik, Technische Universität Wien, Austria
- Karen Dee Michalowicz⁺, The Langley School, VA, and Graduate School, Mathematics Education, George Mason University, VA, USA
- Luis Radford, Ecole des Sciences de l' Education, Laurentian University, Canada
- Gert Schubring, Institut für Didaktik der Mathematik, Universität Bielefeld, Germany,
- Man-Keung Siu, Department of Mathematics, University of Hong Kong, China
- Bjorn Smestad, Faculty of Education, Oslo University College, Norway (Newsletter co-editor)
- Robert Stein, California State University, San Bernardino, USA
- Constantinos Tzanakis, Department of Education University of Crete, Greece (chair)
- Jan van Maanen, Freudenthal Institute, Utrecht, The Netherlands (former chair 1996-2000)
- Chris Weeks, Downeycroft, Virginstow Beaworthy, UK (Newsletter co-editor)

In addition, having appreciated the importance of the *HPM Newsletter* as a basic tool to realize the aims and the associated activities of the Group – especially during the preceding period, chaired by F. Furinghetti - the *Newsletter* has been successfully prepared and edited by three co-editors: B. Smestad (Norway), N. Kastanis (Greece), Ch. Weeks (UK).

2. The HPM Newsletter

In the last few years, the *HPM Newsletter* has evolved into an informative document, published three times a year and distributed worldwide via a network of distributors¹, who are in charge of sending the *Newsletter* either in paper or in electronic form to interested people of a particular region in the world. In addition the *Newsletter* is available on the web through the two main websites of the HPM Group: The main website at <http://www.clab.edc.uoc.gr/hpm/> and the website of the Americas Section of the HPM Group at <http://www.hpm-americas.org/>

Since 2004, each issue of the newsletter has 24 to 28 pages, including some standard sections, namely, (often annotated) recent publications, list of relevant websites, book reviews, presentations of recently completed PhD theses, announcement of events (conferences, meetings, workshops etc), conference and meetings' reports, interviews with leading scholars in this area. In addition there are texts that are devoted to special subjects, in the history of mathematics, the history of mathematics education, or the integration of a historical dimension in mathematics education. Since July 2004, 11 issues have been published (No 57 to No 67) and another one (No 68) will appear in July 2008, immediately before ICME 11.

3. The HPM websites

Information on the HPM Group, its aims, history and activities, together with details on relevant documents and resources can be found in the HPM official website <http://www.clab.edc.uoc.gr/HPM/> and the website of the *Americas Section of the HPM Group* <http://www.hpm-americas.org/> All issues of the HPM Newsletter since 2000 (from No 45 onwards) are available for download. In addition, details on conferences and meetings, as well as links to societies, unions, other groups, resources in several languages, journals etc are also available there.

¹Currently this network consists of 27 distributors in an equal number of regions worldwide. The complete list appears in each issue of the Newsletter.

4. Conferences and Meetings

4.1 The HPM Group at ICME

(a) ICME 10, Copenhagen, Denmark, 4-11 July 2004

Activities of the HPM Group during ICME 10:

-Topic Study Group 17: The role of the history of mathematics in mathematics education

Organizing Team: A. Elidrissi (Morocco), S. Kaijser (Sweden), L. Radford (Canada), M-K. Siu (China, co-chair) and C. Tzanakis (Greece, co-chair).

The work in this group led to the publication of a special issue; see §5.2(a).

-Topic Study Group 29: The History of the Teaching and Learning of Mathematics

Organizing Team: H. Gispert (France), H-C. Hansen (Denmark), H. Khuzwayo (South Africa), G. Schubring (Germany, co-chair) and Y. Sekiguchi (Japan, co-chair).

The work in this group led to the publication of a special issue; see §5.2(b).

-The ASG meetings of HPM in ICME-10

There were three sessions with an equal number of talks discussing the identity of the group and in particular the following points: (i) to make known the origin of the group; which persons contributed to its birth and growing and which was the initial impetus for the formation of this group, (ii) to stress once again the basic ideas underlying the activities of the group, (iii) to outline, the results of work done in the period 2000-04 and the perspectives for the future.

-Regular Lectures related to the HPM Group: There were three such lectures by V. Katz (USA), L. Puig (Spain) and E. Shchepin (Russia).

-Workshops and Sharing Experience Groups (SGA): One workshop (by V. Katz & K.D. Michalowicz⁺ (USA)) presented the material referred to in §5.1(a); one SGA (by A. Gazit (Israel)) was devoted to history in pre-service Mathematics Teachers Education.

-Poster Round Table: Four posters related to HPM issues were discussed in a Round Table.

(b) ICME 11, Monterrey, Mexico, 6-13 July 2008

Activities of the HPM Group during ICME 11

-Topic Study Group 23: The role of history of mathematics in mathematics education

Organizing Team: A. Elidrissi (Morocco), A. Miguel (Brazil), E. Barbin (France), A. Garciadiego (Mexico).

-Topic Study Group 38: The History of the Teaching and Learning of Mathematics

Organizing Team: R. d'Enfert (France), Á. Ruiz (Costa Rica), L. C. Arboleda (Colombia), R. Cambray (Mexico), W-S. Horng (Taiwan).

-The ASG meetings of HPM in ICME-11: There will be two 2-hour sessions with three talks and discussions to report on the work that has been done in the last four years, the future perspectives and ways to arouse further the interest on the activities of the group and increase its visibility. In particular, there will be talks and discussions on

(i) The history of mathematics Education;

(ii) Women's role in Mathematics Education since ICMI was founded;

(iii) Integrating original sources in mathematics teaching;

(iv) Discussing the future of the group with focus on its activities so far, their further development and new possibilities.

-Regular Lectures related to the HPM Group: There will be three such lectures by K. Bjarnadóttir, (Iceland), G. Schubring (Germany) W. Rodrigues Valente (Brazil & Portugal),

-Workshops: Two workshops (by E. de Souza Lodron Zuin and J. C. Barreto Garcia) are related to the HPM interests.

-Poster exhibition: Two posters related to HPM Issues are included in the programme.

4.2 The HPM Satellite Meetings of ICME

(a) HPM 2004 & ESU 4 (ICME 10 Satellite Meeting of HPM & 4th European Summer University on History and Epistemology in Mathematics Education), Uppsala, Sweden, 12-17 July 2004

Main Themes:

1. The history of mathematics
2. Integrating history of mathematics into the teaching of mathematics
3. The role of the history of mathematics in teacher's training
4. The common history of mathematics, science and technology
5. Mathematics and different cultures
6. The philosophy of mathematics

Structure of The Program

1. Six plenary sessions, including: 6 invited lectures (one per day), 2 Panel discussions
2. Sessions consisting of: 9 workshops (from one to two hours), 59 paper presentations (in 3 parallel sessions).

Participation: 120 participants from 32 countries

(b) HPM 2008 (ICME 10 Satellite Meeting of HPM 11), Mexico City, 14-18 July 2008

Main Themes:

1. Integrating the History of Mathematics in Mathematics Education.
2. Topics in the History of Mathematics Education.
3. Mathematics and its relation to science, technology and the arts: historical issues and educational implications.
4. Cultures and Mathematics.
5. Historical, philosophical and epistemological issues in Mathematics Education.
6. Mathematics from the Americas

Structure of The Program (not finalized yet)

1. Four plenary sessions, including: 6 invited lectures (one per day), 1 Panel discussion
2. Sessions consisting of: 6 one-hour workshops, 84 paper presentations (in 4 parallel sessions).

Participation: Over 150 participants from 30 countries

4.3 The European Summer University on History and Epistemology in mathematics Education (ESU)

(a) ESU 4 took place conjointly with HPM 2004; see §4.2(a) above.

(b) ESU 5 (5th European Summer University on History and Epistemology in Mathematics Education), Prague, Czech Republic, 19-24 July 2007.

Main Themes

1. History and Epistemology as tools for an interdisciplinary approach in the teaching and learning of Mathematics and the Sciences
2. Introducing a historical dimension in the teaching and learning of Mathematics
3. History and Epistemology in Mathematics teachers' education
4. Cultures and Mathematics
5. History of Mathematics Education in Europe
6. Mathematics in Central Europe

Structure Of The Program

1. Six plenary sessions, including: 6 invited lectures (one per day), 2 Panel discussions
2. Sessions consisting of: 19 2-hour workshops (based on didactical and pedagogical material), 25 3-hour workshops (based on historical and epistemological material), 44 oral presentations and 26 short announcements, in six parallel sessions.

Participation: 192 participants from 33 different countries.

4.4 HPM in CERME

CERME 6: Université de Lyon 1 (France) January 27 - February 1, 2009. Website: <http://ermeweb.free.fr/>

A Working Group (WG 15) entitled *The Role of History of Mathematics in Mathematics Education: Theory and Research* has been included in CERME 6 (Congress of the European Society for Research in Mathematics Education). Work in this context is under progress.

The purpose of this Working Group is to provide a forum primarily dedicated to theory and research on the role of history in mathematics education. It is particularly interested in theoretical and empirical studies (including work-in-progress) that address one or more of the following themes:

- Theoretical and/or conceptual frameworks for including history in mathematics education.
- The role of history of mathematics at primary and secondary level, both from the cognitive and affective points of view.
- The role of history of mathematics in pre- and in-service teacher education, both from the cognitive, pedagogical, and affective points of view.
- Possible parallelism between the historical development and the cognitive development of mathematical ideas.
- Ways of integrating original sources in classrooms, and their educational effects, preferably with conclusions based on classroom experiments.
- Surveys on the existing uses of history in curriculum, textbooks, and/or classrooms in primary, secondary, and university levels.
- Design and/or assessment of teaching/learning materials on the history of mathematics.

Organizers of the group: F. Furinghetti (Italy), U. Th. Jankvist (Denmark), C. Tzanakis (Greece), J-L. Dorier (France)

4.5 Regional and other meetings pertaining to HPM

(a) “History of Mathematics, History of Mathematics Education and their didactical implications”, 14-15 April, 2006 Department of Mathematics, University of Thessaloniki, Thessaloniki, Greece.

Motivated by the activities and results of the ICME 10 Topic Study Groups TSG 17, and TSG 29, Greek researchers interested in the HPM perspective, organized a regional meeting at the University of Thessaloniki, Greece, aiming to present current international trends on important issues relevant to the HPM perspective, with reference to all educational levels, and in this way to give an opportunity to the Greek educational community to be informed about those issues that are of great importance nowadays.

(b) “Mini-Workshop: Studying Original Sources in Mathematics Education”, Mathematisches Forsschunsinstitut Oberwolfach, Germany, 1-5 May 2006. Organisers F. Furinghetti (Italy), H. N. Jahnke (Germany), J. van Maanen (The Netherlands).

This Workshop aimed at investigating the benefit that the teaching and learning of Mathematics may have on the basis of studying original texts; in particular it made focus on some ideas, which are specifically supported by reading mathematical sources:

- to see mathematics as an intellectual activity;
- to place mathematics in the scientific, technological and philosophical context of a particular period in the history of ideas;
- to participate in an activity oriented more to processes of understanding, than to final results;
- to appreciate the role and importance of the different languages involved; those of the source, of modern mathematics and of everyday life;
- to see what is supposed to be “familiar”, becoming “unfamiliar”.

There have been 17 contributions from 16 contributors coming from 10 different countries, who gave detailed presentations on particular cases concerning the points mentioned above. These presentations were followed by lively discussions, in which participants were given the opportunity to elaborate on

their ideas further. Each presentation and the follow-up discussion were based on material (original texts, students' worksheets, etc) distributed in advance, or on the spot.

5. Publications

5.1 Proceedings of Conferences and other collective volumes

(a) *Historical Modules for the Teaching and Learning of Mathematics* Editors: V. J. Katz (USA) & K. D. Michalowicz (USA), The MAA, Washington DC, 2005 (in electronic form).

This is an outcome of the "Historical Modules Project", co-directed by V. Katz and K. D. Michalowicz, a special activity of the "Institute in the History of Mathematics and its Use in Teaching" (IHMT)², where about thirty high school and college teachers of mathematics teamed up to produce this didactical material organized in 11 modules. For a recent review see G.E. FitzSimons, "Review of Historical Modules for the Teaching and Learning of Mathematics" *Australian Senior Mathematics Journal*, **20** (2), 62-64, 2006.

(b) *Proceedings of HPM 2004 & ESU 4* Editors: F. Furinghetti (Italy), S. Kaisjer (Sweden), C. Tzanakis (Greece), University of Crete, Greece, 2006. ISBN 960-88712-8-X (676 pages).

This is the revised edition of the Proceedings of the HPM Satellite Meeting of ICME 10 and the 4th European Summer University on the History and Epistemology in Mathematics Education, which took place co-jointly in Uppsala, Sweden, in 2004 (see §4.2(a) above). It consists of 78 papers divided into 6 sections, corresponding to the 6 main themes of this meeting

(c) *History and Mathematics Education* Editors: Y. Thomaidis, N. Kastanis, C. Tzanakis. Ziti Publications, Thessaloniki, Greece 2006 (287 pages, in Greek). ISBN 960-431-997-3.

This volume consists of the Proceedings of the regional meeting of the HPM Group of §4.5(a). It includes 14 papers divided into three sections: (i) On the History of ancient Greek Mathematics, (ii) On the History of Mathematics Education (iii) On the relations between History of Mathematics and Mathematics Education.

(d) *History and Epistemology in Mathematics Education: Proceedings of the Fifth European Summer University (ESU 5)* Editors: E. Barbin (France), N. Stehlikova (Czech Republic), C. Tzanakis (Greece), Vydavatel'sky servis, Plzeň, Prague, Czech Republic, 2008. ISBN 978-80-86843-19-3 (902 pages).

This volume consists of 120 peer reviewed papers and abstracts, based on the activities during ESU 5 (see §4.3(b)), divided into six sections corresponding to the six main themes of this Summer University. This volume is expected to appear in summer 2008 and will also be available on the web.

5.2 Special issues of scientific journals related to the HPM perspective

(a) "The role of the History of Mathematics in Mathematics Education" *Mediterranean Journal for Research in Mathematics Education* special double issue, vol 3, nos 1-2, 2004 (166 pages). Guest Editors: M-K. Siu (China), C. Tzanakis (Greece)

This is a special double issue based on peer-reviewed papers that have been originally presented in TSG 17 (The role of the History of Mathematics in Mathematics Education) at ICME 10. There are 10 papers divided into 4 sections: (i) Epistemological issues, (ii) Teachers' education, (iii) Didactical material, (iv) Particular Examples.

(b) "History of Teaching and Learning Mathematics" *Paedagogica Historica. International Journal of the History of Education* vol. XLII, nos IV&V, August 2006. Guest Editor : G. Schubring (Germany).

² For more details on IHMT, see the HPM Newsletter, No 62 (2006), pp.17-21. Two of the three directors of IHMT have been active members of the HPM; V. Katz, University of the District of Columbia and F. Rickey, now of the United States Military Academy, West Point, New York. Another member of the Group and one of its former chairs, F. Fasanelli served as MAA liaison and was instrumental in obtaining the National Science Foundation grants that funded the Institute.

This is a special double issue based on peer-reviewed papers that have been originally presented in TSG 29 (The history of the Teaching and learning of Mathematics) at ICME 10. There are 9 papers divided into three sections: (i) Transmission and Modernizations of Mathematical Curricula, (ii) Teaching Practice, Textbooks, Teacher Education (iii) Cultural, Social and Political Functions of Mathematics Instruction.

(c) “History of Mathematics in Mathematics Education: Theory Practice.” *Educational Studies in Mathematics*, special issue, vol.66, no2, 2007 (164 pages). Guest Editors: F. Furinghetti (Italy), V. Katz (USA), L. Radford (Canada).

This is a special issue, which consists of 10 peer-reviewed papers. According to the editors, “the papers seek to deepen our understanding of the pedagogical role that the history of mathematics may play in contemporary mathematics education. Some of the papers provide examples of the use of the history of mathematics in school practice and in teacher education. Other papers address theoretical questions that have become crucial to understanding the profound intertwining of past and present conceptual developments from spreading new epistemologies and theories of learning” (p.109).

5.3 Scientific journals/bulletins/newsletters pertinent to the HPM perspective

(a) *The “HPM Tongxun” and the Tongxun Group in Taiwan*. Editor: W-S Horng, Department of Mathematics, National Taiwan Normal University, Taiwan.

This is a publication in the context of the HPM Group, published in Chinese since 1998 on a monthly basis. Contributors to this Newsletter are mathematics teachers, postgraduate students or researchers in this area. This collaboration encourages them to promote the HPM activities – some of them even become local leaders for both the HPM and mathematics teaching. They become very enthusiastic about the HPM perspective, contributing to this Newsletter just to share their ideas and vision with their colleagues. Despite the fact that no formal organization, such as a society, is to be expected for “up-scaling” development, a total of about forty correspondents plus ten editorial members is a strong indication that an active local HPM group could emerge.

For details see <http://www.math.ntnu.edu.tw/~horng>

(b) *International Journal for the History of Mathematics Education*. Chief Editor G. Schubring (Germany), Managing Editor: A. Karp (USA/Russia), published by COMAP Inc. USA. (two issues per year, available in printed form and online).

The rousing success of the Topic Study Group 29, *The History of Learning and Teaching Mathematics*, at ICME 10 in Copenhagen in 2004, demonstrated the need for a permanent and stable international forum for scholarly research in history of mathematics teaching. Therefore, a new journal has been established. The first issue appeared in September 2006. The journal is supported by Teachers College, Columbia University, USA. Its website is <http://www.tc.edu/centers/ijhmt/>

(c) *The Bulletin of the British Society for the History of Mathematics (BSHM Bulletin)*

Published as a Newsletter until 2004 when its 50th issue became Bulletin 1. Under the influence of the late John Fauvel, who was President of BSHM from 1992–1994, editor of its Newsletter from 1995–2001, and chair of the HPM Group from 1992–1996, the Newsletter changed from being an information advice to members to a publication of serious articles. This change strengthened under Jackie Stedall who took over as editor in 2002. The Bulletin is now much more of a scientific journal particularly since 2004 when it started to be published by Taylor & Francis three times a year. Since the summer of 2002 (issue No 46) the Bulletin has had a regular Education Section, directly related to issues relevant to the HPM interests and perspective.