International Study Group on the Relations Between the HISTORY and PEDAGOGY of MATHEMATICS
An Affiliate of the International Commission on Mathematical Instruction

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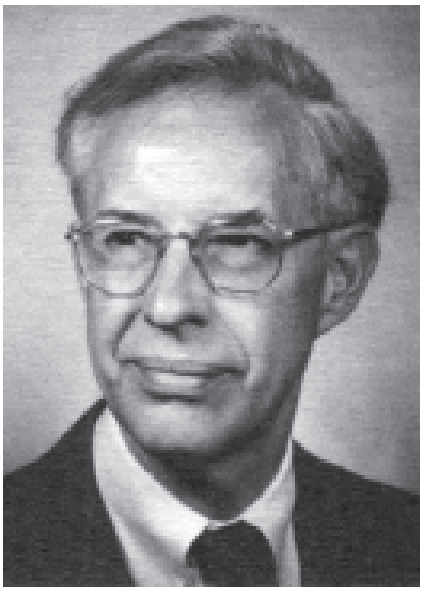
http://www.clab.edc.uoc.gr/hpm/

Phillip S. Jones (1912-2002)

One of the HPM pioneers

It is known that the modern idea of the *Relations Between the History and Pedagogy of Mathematics* (HPM) came about 1972, during the 2nd ICME, which was held in Exeter in the UK. It is also known that HPM group was created by the ICMI-committee in 1976 as one of its Study Group with the first co-chairs: P.S. Jones and R. Stowasser. But very little is known about Phillip S. Jones. Now is a good time for the new generation of HPM group's participants to learn about one of its pioneers.



Jones graduated in mathematics with bachelor's (1933) and master's (1935) degrees from the University of Michigan. His paper on *Early American Geometry* appeared in *The Mathematics Teacher* of 1944 [vol. 37, pp. 3-11], the first of his publications, and was perhaps the paper that opened the way for his academic career.

After a decade of teaching mathematics at high school and college level, Jones returned to University of Michigan and received a PhD in 1948. His dissertation was: *The Development of the Mathematical Theory of Linear Perspective and Its Connections with Projective and Descriptive Geometry with Especial Emphasis on the Contributions of Brook Taylor*, supervised by Professor Louis C. Karpinski (1878-1956), the prominent American historian of mathematics and educator. Jones joined the University of Michigan faculty as an instructor in 1947, and became a professor in the Mathematics Department in 1958. His areas of speciality were mathematics education and the history of mathematics

In his 1948 report on the University of Michigan project for the teacher-training of future mathematics instructors and professors, Jones pointed out that one topic of interest was: "the nature, sources, and the uses of enrichment materials [*of mathematics*] (e.g. historical ideas, models, applications or connections with other fields)". And for this reason, he proposed the use of "historical works by D. E. Smith, F. Cajori, and R. C. Archibald" [*The American Mathematical Monthly*, Vol. 55, No. 3. (Mar., 1948), pp. 145-147].

Beginning in 1951, Jones organized an annual Mathematics Education Conference, held at the University of Michigan. Mathematics teachers and school administrators from across the state and country attended the conference, and it developed into the largest and most influential conference on mathematics education in the state. In the same year his paper on *Brook Taylor and the Mathematical Theory of Linear Perspective* [*The American Mathematical Monthly*, Vol. 58, No. 9. (Nov., 1951), pp. 597-606] appeared.

The development of his publications in the fifties is very interesting because it was then that his contributions in the pedagogical uses of the history of mathematics and in the history of American mathematics education had begun to bear fruit. The following papers bear witness to his work:

1. "The Pythagorean Theorem", in *The Mathematics Teacher*, 43, 1950,
2. "Miscellanea — Mathematical, Historical, Pedagogical," *The Mathematics Teacher*, 43, 1950,
3. "Angular Measure-Enough of Its History to Improve Its Teaching", in *The Mathematics Teacher*, 46, 1953,
4. "The Binary System", in *The Mathematics Teacher*, 46, 1953,
5. "Complex Numbers: An Example of Recurring Themes in the Development of Mathematics-I,II,III", in *The Mathematics Teacher*, 47, 1954,
6. "Word Origins", in *The Mathematics Teacher*, 47, 1954,
7. "Tangible Arithmetic, I-Napier's and Genaille's Rods", in *The Mathematics Teacher*, 47, 1954,
8. "America's First Mathematicians", in *The Mathematics Teacher*, 48, 1955,
9. "Tangible Arithmetic, IV-Finger Reckoning and Other Devices", *The Mathematics Teacher*, 48, 1955,
10. "Louis C. Karpinski, historian of mathematics", *Science*, 124, 1956,
11. "The History of Mathematics as a Teaching Tool", in *The Mathematics Teacher*, 50, 1957,
12. "Films and Television: Notes on the Status and Future of Films and TV in Collegiate Mathematics Education", *The American Mathematical Monthly*, 65(6), 1958,
13. "The Growth and Development of Mathematical Ideas in Children", in *The Growth of Mathematical Ideas. Grades K-12*, the 1959 NCTM Yearbook, and
14. (with G. Gibb and Ch. W. Junce) "Number and Operations", in *The Growth of Mathematical Ideas. Grades K-12*, the 1959 NCTM Yearbook.

His contributions to the area of the education of mathematics teachers and its relations with the history of mathematics have laid the foundations for his future academic activities. He was a founder of the Michigan Council of Teachers of Mathematics, a member of the board of governors of the Mathematical Association of America and a member of the US Commission on Mathematics Instruction appointed by the National Academy of Science.

Phillip S. Jones was the Chair of the Mathematical Association of America Committee on Films for Classroom Instruction from 1955-1959 and the NRC Film Evaluation Board and Divisional Committee on Mathematical Films and Television (1957-1958). He was a member of the board of governors of the Mathematical Association of America, and a member of the US Commission on Mathematics Instruction appointed by the National Academy of Science. He also served as a director of the National Council of Teachers of Mathematics (NCTM) from 1953-1959, as its vice president from 1959-1960 and as its president from 1960-1962.

During that time one of his major publications, *The Growth of Mathematical Ideas, K-12*, appeared as the 1959 NCTM Yearbook. A main thesis of the book is that students should continually have recurring but varied contacts with the fundamental ideas and processes of mathematics — number and operation; relations and functions; proof; measurement and approximation; probability; statistics; language and symbolism of mathematics; and mathematical modes of thought. This view of the curriculum has been broadly accepted in elementary and middle schools and has influenced secondary school curricula.

In the 1960s the following papers of his were published:

1. "Recent Research in Mathematics: Implications for Teacher Education", *The American Mathematical Monthly*, 67(6), 1960,
2. "A History of Attempts to Improve School Mathematics in the United States", in *Report of a Conference on Future Responsibilities for School*

Mathematics, School Mathematics Study Group, 1962.
3. (with. A. F. Coxford, Jr.) "Academic and Professional Preparation of Secondary School Mathematics Teachers", in *Review in Educational Research*, 34, 1964,
4. "The History of Mathematical Education**",** in *The American Mathematical Monthly*, 74, 1967,
5. "The History of Mathematics as a Teaching Tool", in *Historical Topics for the Mathematics Classroom*, the 1969 NCTM Yearbook, and
6. "Angular Measure", in *Historical Topics for the Mathematics Classroom*, the 1969 NCTM Yearbook.

In 1970, Phillip S. Jones collaborating with Arthur F. Coxford, Jr. edited the 32nd NCTM Yearbook on *A History of Mathematics Education in the United States and Canada.* It was a well-organized effort to identify the influence of the widespread changes taking place in mathematics education in the United States and Canada. The editors and the authors of the fourteen chapters made an outstanding contribution by compiling the history of these changes and the book records the sources North America's evolution in mathematics education, with emphasis on the relevant committees, commissions, conferences, symposiums, panels, et al.

This book contributed to the awakening of a modern interest for the history of mathematics education, and in the recognition or legitimisation of the historical studies in this field. It was a significant step toward the international development of historical activities of this kind. A step which Jones gradually promoted in the sixties, and which was realised in the period of expansion of ideas and associated conflicts emerging from the Modern Mathematics Reform.

The reaction to 'modern mathematics' was clearly in evidence at the 2nd International Congress of Mathematics Education (1972) in Exeter, UK. There it opened a new epistemological spirit and trends in the understanding of the mathematics education. By the way, it is famous for the statement of René Thom (1923-2002), in this congress, that "all mathematical pedagogy, even scarcely coherent, rest on a philosophy of mathematics", which supports the view that mathematics teaching and learning is not only a question of establishing a formalistic behaviour, but also of forming a common meaning. This was instrumental in helping to establish the international movement of pedagogical uses of history of mathematics. Jones was at Exeter and played a creative role in the impetus of this movement, together with David Wheeler, A. G. Howson, et al. and so it was natural for him, together with R. Stowasser, to take on the chair of the HPM group when it was formed in 1976.

In the period 1970-1990 Jones published the following:

1. (with. A. F. Coxford, Jr.) "The Goal of History: Issues and Forces" in *A History of Mathematics Education in the United States and Canada*, the 1970 NCTM Yearbook,
2. (with. A. F. Coxford, Jr.) "Mathematics in the Evolving Schools" in *A History of Mathematics Education in the United States and Canada*, the 1970 NCTM Yearbook,
3. "Present-Day Issues and Forces" in *A History of Mathematics Education in the United States and Canada*, the 1970 NCTM Yearbook,
4. "Historical Background and Founding of the Association", in *The Mathematical Association of America: Its First Fifty Years*, The Mathematical Association of America, ed. by May, K.O., 1972,
5. "A role for the History of Mathematics", in *Historia Mathematica*, 2, 1975,
6. "Louis Charles Karpinski, Historian of Mathematics and Cartography", in *Historia Mathematica*, 3, 1976,
7. (with Bunt, L.N.H. and Bedient, J.D.) "*The Historical Roots of Elementary Mathematics*", Prentice-Hall Publ., 1976,
8. "Bibhutibhusan Datta. A Note and a Question", in *Historia Mathematica*, 3, 1976,
9. "The History of Mathematics as a Teaching Tool" in Zentalblatt für Didaktik der Mathematik, 10(2), 1978,
10. "Motivations and Forces Effecting Changes in Mathematics Education in the Nineteenth Century in the United States", in *Epistemologischen und soziale Problem der Wissenschaftsentwicklung im frühen 19. Jahrhundert*, IDM, Bielefeld, 1979,
11. "The history of mathematics---new sources and uses", in *Southeast Asian Bulletin of Mathematics*, 4(1), 1980,
12. "The development of Mathematics Education as a professional Discipline in U.S.A.", in Proceedings of ICME IV, Birkhäuser, ed. By Zweng, M. et al., 1983, pp. 485-486, and
13. "The role in the history of mathematics of algorithms and analogies", in *Learn from the masters!*, *Proceedings of the Kristiansand Conference on the History of Mathematics and its Place in Teaching*, ed. by Bekken, O. et al 1988.

Jones retired from the University of Michigan faculty in 1982. He continued to pursue his interests and related activities in the eighties, as can be seen with his participation in the 1988 Kristiansand (Norway) meeting of HPM. In retirement he had more opportunity to share with his wife his family of four children, eight grandchildren; and four great-grandchildren.

Phillip S. Jones died on June 27, 2002 at Ann Arbor, Michigan.

Phillip S. Jones belonged, together with C. Boyer, M. Kline, H. Eves, E.T. Bell, D. Struik et al., to the intermediate generation of American historians of mathematics which continued the impetus of his co-patriot pioneers in the field [like F. Cajori, D.E. Smith, R.C. Archibald, L.C. Karpinski et al.] and left his heritage to the American renewal of the history of mathematics in the hands of our contemporaries [like J. Grabiner, L. Daston, J. Richards, J.W. Dauben, F. Rickey, V. Katz et al.].

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Conference reports

The editors welcome reports from conferences.

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Work in progress

We encourage young researchers in fields related to *HPM* to send us a brief description of their work in progress or a brief description of their dissertation.

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Ph.D. thesis

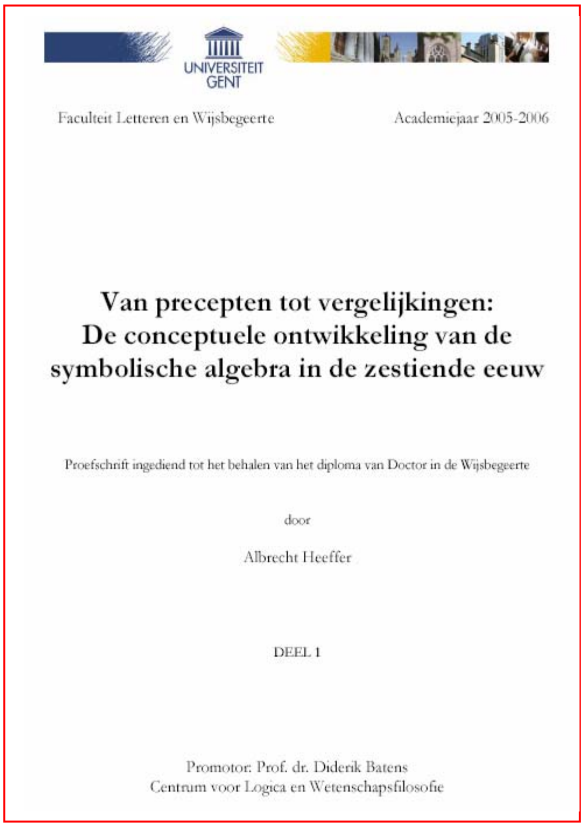
Albrecht Heeffer: Van precepten tot vergelijkingen:

De conceptuele ontwikkeling van de symbolische algebra in de zestiende eeuw
[From Precepts to Equations, the Conceptual Development of Symbolic Algebra in the Sixteenth Century]
Ghent University, Belgium, 2006

The following is the concluding summary from Dr Albrecht Heeffer's dissertation.

Concluding this dissertation we should account for our preoccupation with problem solving throughout the previous pages. The first rationale stems from Piaget's central idea of interiorization. Knowledge results from action and mathematical practice consists mainly of problem solving. Hence, new concepts emerge as an interiorization or solidification of repeated actions and operations. 'Action' has here a double meaning. On the one hand it refers to the practice of arithmetical problem solving. The emergence of new concepts, such as the symbolic equation, follows many years of practice of manipulating and equating polynomial expressions in one or multiple unknowns. Changes in the meaning of existing concepts, such as the broadening of the number concept, result from solving many types of problems over centuries. On the other hand, 'actions' also refer to specific mathematical operators such as multiplication or root extraction.

In contrast with Jacob Klein (1934-6), rooted in German idealism, we do not regard concepts as living a life on their own. We have demonstrated that the concepts of early symbolic algebra are abstractions of comprised operators, with a specific function in algebraic problem solving. Viewing mathematical practice as problem solving is evident from our corpus of historical sources. Most of the abacus texts only deal with problems and their solutions. As argued in chapter 8 [*The Rhetoric of Problems in Algebra Textbooks from Pacioli to Euler*], algebraic theory as a collection of theorems, finds its way into textbooks only from the seventeenth century. But even the characterization of modern mathematics as problem solving is fruitful for an understanding mathematical practice. Modern mathematical practice such as conceiving proofs for theorems or conjectures involves a variety of activities, which are mostly problem-solving oriented. The historical study of arithmetical problem solving touches therefore the core of mathematical activities and provides us with epistemological insights into mathematics.



UNIVERSITEIT GENT

Faculteit Letteren en Wijsbegeerte — Academiejaar 2005-2006

Van precepten tot vergelijkingen: De conceptuele ontwikkeling van de symbolische algebra in de zestiende eeuw

Proefschrift ingediend tot het behalen van het diploma van Doctor in de Wijsbegeerte

door

Albrecht Heeffer

DEEL 1

Promotor: Prof. dr. Diderik Batens
Centrum voor Logica en Wetenschapsfilosofie

A second motivation for our focus on problem solving is related to our research strategy. We have chosen to follow a bottom-up approach, from actual practice to reflection and generalization. An understanding of the way the abacus and reckoning masters conceive the objects of their analysis and the conceptualizations involved, requires an in-depth mastery of the techniques and methods employed in solving problems. It took about two years to get the grip on this recurring process of rhetorical problem solving and to deliver the necessary insights and recognizing skills. While working on problems with numbers in GP [*geometric progression*], the opportunity to reconstruct a solution to a "missing" problem class proved to be intensively gratifying. The fact that one obvious class of problems does not occur in the literature appears to be an odd coincidence. We would suspect that a top-down approach would necessitate a fabricated explanation while experience with this type of problem solving allows us to reconstruct a solution in typical Cardanian style.

A third reason to concentrate on problems is the role of problem solving as a social, cultural and even economical activity. We have placed mathematical practice in the context of the society in which it functions and consequently we have related historical changes in mathematical conceptualizations to structural changes in that society. It is not always possible or desirable to relate the evolution of mathematics with social change but the case of symbolic algebra is intriguing in this respect. The emergence of a symbolic mode of reasoning was probably the most important evolution in the mathematical sciences since the Greek concept of a mathematical proof.
[From the General Conclusion, pp.563-564]

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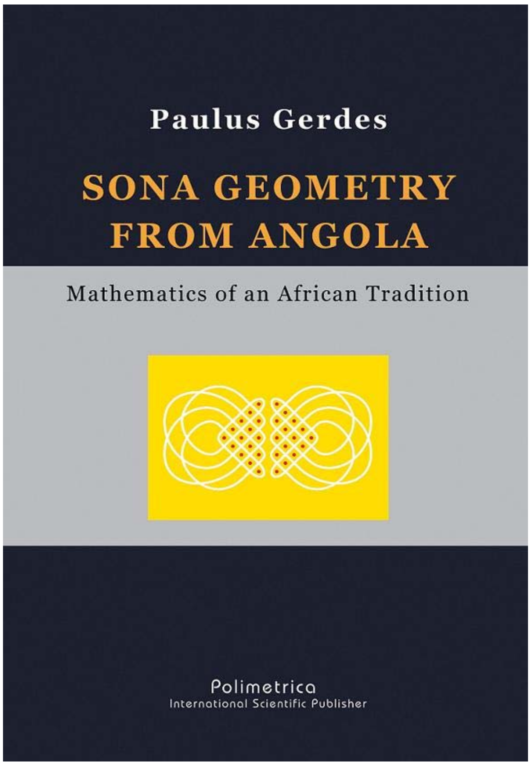
Books

Paulus Gerdes: *Sona Geometry from Angola. Mathematics of an African Tradition*,
Polimetrica Scientific Publisher, Monza, November 2006

From the preface by Professor Arthur B. Powell:
This volume provides readers with a glimpse into Paulus Gerdes's seminal work on the mathematics of an African tradition—*sona* geometry, a drawing and narrative tradition from Angola with embedded mathematical ideas. The work represented in this book contributes significantly to efforts by other

African mathematicians and mathematics educators to recuperate and valorize mathematical ideas and reasoning that reside in African material culture and cultural practices. Moreover, Gerdes is a prolific contributor of work that reinforces a growing literature available in English of a dynamic research program in ethnomathematics…



Uncovering the mathematical ideas embedded in a Cokwe cultural practice and providing access to the richness of these ideas are part of the concerns of ethnomathematics. As an ethnomathematician, Gerdes is not a neutral researcher but rather a public intellectual committed to finding ways not only to understand the mathematics of the Cokwe *sona* tradition but also to raise important questions such as one he hints at in the Epilogue when he observes that the profoundness of the mathematical ideas expressed in the *sona* "had started to build up." With this, his reader is then prompted to ask questions: What happened to the building up of the *sona* tradition and its mathematical ideas? Why was this development arrested? Answering these questions along with enjoying the beautiful presentation of the mathematics of the *sona* tradition are the gifts that readers will receive from this wonderful volume.

Doing mathematics with historical documents

Demattè, A. (editor), 2006, ***Fare matematica con i documenti storici – una raccolta per la scuola secondaria di primo e secondo grado. a - Volume per gli studenti, b - Volume per l'insegnante***, Editore Provincia Autonoma di Trento – IPRASE del Trentino (Italy).

A. Demattè, the editor of 'Doing mathematics with historical documents' explains the structure and philosophy of the work:
I wish to present a book, of which I am editor, entitled *Fare matematica con i documenti storici*. The book brings together a collection of passages selected from original sources. As highlighted by the words '*Fare matematica*' ('Doing mathematics') in the title, it is not so much a resource for 'reading about mathematics' but rather for working with problems and exercises. The aim of this book is to provide secondary school teachers with suggested activities to integrate original sources into everyday classroom work. This integration should promote alternative ways of teaching through text-based activities and exercises to consolidate (or sometimes even to introduce) mathematical skills.

The genesis of the book

This brief anthology of documents aimed at secondary school students (aged 12-18) is the result of two years of work carried out by five in-service teachers who opted to collaborate with IPRASE - *Istituto Provinciale di Ricerca, Aggiornamento e Sperimentazione Educativi del Trentino* (Institute of the Province of Trento, Italy, for research, training and experimentation in the field of education). IPRASE does not focus specifically on research in the field of mathematics but aims to improve the quality of schooling in this alpine province with a population of 400,000. Consideration was given to the educational potential of the history of mathematics and the use of original sources in the classroom. The principal objective of our small group of teachers was to produce materials for everyday activities in the classroom. The main source for our work was an Italian publication, Bottazzini, Freguglia & Toti Rigatelli, *Fonti per la storia della matematica* (Sources for the history of mathematics), a collection of documents regarding arithmetic, algebra, geometry, calculus, logic and probability. The documents selected for inclusion in the new book include writings by important authors as well as by lesser known mathematicians whose works were representative of their time. Reference is made to the main topics taught in Italian secondary school.

Activities for students

In *Fare matematica con i documenti storici* each document is followed by questions and activities whose purpose is to focus students' attention on the main points in the original source through active engagement. Some questions involve follow up exercises and problems, giving different examples or making reference to textbook topics.
An original source may include text, diagrams and formulas, reflecting the many forms of communication in mathematics. The cultural context becomes apparent from the documents implicitly, as a necessary fact, without the need for specific descriptions. Although they will have had no previous formal teaching of the history of mathematics, students will nevertheless be able to investigate the origins of mathematical ideas. As mentioned earlier, the aim of the book is to provide secondary school teachers with suggested activities to integrate original sources into everyday classroom work, not as 'paradigmatic' experiences but as consolidation tasks, for instance, medieval algorithms for arithmetic operations, or occasionally to introduce a new idea, such as topology. The activities that follow each document also help the student gain a better understanding of mathematical ideas, such as ancient numeration systems, and improve their skills in critical analysis, for example, identifying inaccuracies such as Boole's repeated use of an adjective. For the most part these activities are based on text analysis although students are sometimes asked to reflect on the causes of certain historical facts.

In Italy, many students have difficulty using textbooks for mathematics. The questions and activities in our book not only help students analyse content but also introduce them to the use of a textbook.

Original sources as a teaching resource

The teaching goals underlying the choice of topics in the book can be summarised by the slogan: 'One more historical document, one less repetitive exercise'. However, not all teachers would agree with this slogan and to take this into account several exercises have been included in the book, some from *Algebra*, the seminal work by Italian mathematician Rafael Bombelli (1530–1573), presenting simple tasks which can be solved by equations.
Original sources sometimes provide students with an alternative route to results already within their reach, Al-Khwarizmi's quadratic equation resolution concerning 'squares and roots equal to numbers', for instance, and the example represented by the equation $x^2+10x=39$. The idea of associating questions for students with an original source document led to the formulation of test questions for the final written mathematics examination in secondary junior school, sat by students aged 13-14. The original sources provide the input whereas the associated exercises relate to mathematical concepts covered during the year.

'Parochial nationalism' could arise in connection with the fact that mathematics has a socio-cultural origin and that people tend to have a preference for the various aspects of the civilisation in which they live. The use of historical documents helps raise awareness among Italian students that mathematics developed all over the world. In fact, the documents in our book are the work of 23 Italian authors, 38 authors from elsewhere in Europe and 14 non-European authors. The latter is significant when compared with the number of foreign authors who appear in the majority of literature texts in use in Italian schools.

The small *Volume per gli insegnanti* specifically addresses teachers and provides teaching suggestions, answer keys, topics for further study and a bibliography. Both the student and teachers' books can be used as teacher-training resources.

A didactic proposal structured around original sources challenges the reasons given by teachers to explain their reluctance to use history ('I don't have time for it in class', 'It's not mathematics', 'How can you set test questions?', etc.). Although reading original sources is a time-consuming and complex task requiring considerable linguistic and mathematical skill, it is not a waste of time since it refers to high-level goals. There *is* however a danger related to the difficulty of interpreting original sources, which could prove to be an insurmountable obstacle for weaker students. In some case the documents presented in the book are copies of original texts, but in most cases they are translations. The difficulties can therefore be reduced although the basic effects of studying a source (*replacement* and *reorientation*) remain.

'Integrating history in mathematics replaces the usual with something different […] making the familiar unfamiliar. […] History reminds us that these concepts were invented and that this did not happen all by itself' (Jahnke, H.N., 2000, 'The use of original sources in the mathematics classroom' in Fauvel, J. & van Maanen, J. (eds.): 2000, *History in mathematics education: the ICMI study,* Kluwer Academic Publishers, Dordrecht, pp. 291-292).

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Metáforas que nos piensan. Sobre ciencia, democracia y otras poderosas ficciones

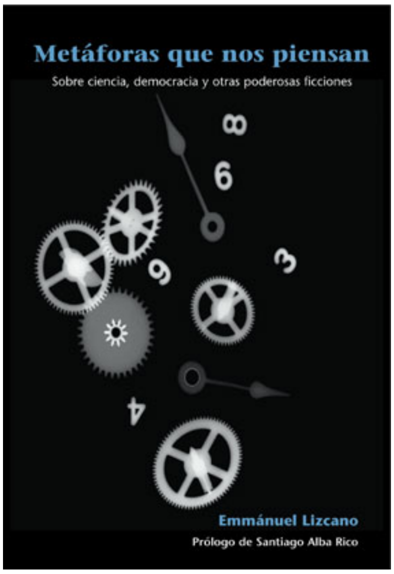
Emmánuel Lizcano

Madrid: Eds. Bajo Cero / Traficantes de Sueños, 2006.

Emmánuel Lizcano describes the background to his book:

Many concepts of ordinary language have a metaphorical origin. Spanish or French speakers talk literally about "to *have* reason" ("to be right") or "to *get out* reason (to somebody)" "to disaffirm (somebody)". They - we?- speak about reason as if it were an object that can be possessed or can be lost. And we argue as if we were fighting about keeping the property over the object (reason) or taking it away from someone. How would we argue, if we thought of reason in terms of a river or as a building under construction?

Many mathematical concepts also have a metaphorical origin. Ancient Greek mathematicians could not conceive of negative numbers because they subtracted numbers *as if* they were 'extracting' (*aphaíresis)* pebbles from a bag: once you have extracted them all, you cannot keep subtracting. Ancient Chinese mathematicians, however, were able to conceive of negative numbers because they didn't *subtract* numbers: they *opposed* numbers. They played them *as if* two armies were facing each other; one of them made up of red sticks and the other one of black sticks, trying to destroy each other (*xiang xiao*): the winner (the result of the "subtraction") is the one that had at the end most warriors, either red ones (*zheng,* positive) or black ones (*fu*, negative).



Metáforas que nos piensan. Sobre ciencia, democracia y otras ficciones makes an approach to that metaphorical constitution of our concepts by proposing numerous examples taken from the mathematical field (‘square *root*’, ‘*natural* numbres’, ‘*real* numbers’, …). On sensing how mathematical language sprouts out from inside ordinary language, the mathematicians get close to poetry and reveal its social and cultural roots (not necessarily ‘*square* roots’). Thus, the chapter on ‘The mathematics of the European tribe: a case study’ tests an ethno-mathematical approach of curricular mathematics. From ethno-mathematics also stems the approach to gipsy arithmetic in ‘Las cuentecitas de los pobres’, ‘Crítica del saber culto y matemática paradójica en el cante flamenco’ and the confrontation between Sioux geometry and that of university professors in ‘Dos invenciones geométricas de lo natural’.

Emmánuel Lizcano has a B.A. in mathematics, a PhD. in Philosophy and is lecturer of sociology at the Spanish Universidad Nacional de Educación a Distancia. He has also written *Imaginario Colectivo y creación matemática. La construcción social del número, el espacio y lo imposible en China y en Grecia*, Barcelona: Gedisa, 1993. *Metáforas que nos piensan* is also available at http://www.bajo-cero.org/ediciones/pdf/lizcano_web.pdf

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Proceeding of HPM-2004 & ESU-4

This is the revised edition of the Proceedings of the HPM satellite meeting of ICME 10 and the Fourth European Summer University on the History and Epistemology in Mathematics Education, which took place cojointly in Uppsala, Sweden, in 2004. The book is entitled *Proceedings of HPM 2004 & ESU 4*, Iraklion, Greece: University of Crete, 2006, ISBN 960-88712-8-X edited by F. Furinghetti, S. Kaisjer and C. Tzanakis. For more information on how to get the book, please contact C. Tzanakis (tzanakis@edc.uoc.gr).

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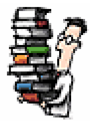

Have you read these?

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* * *

Notices

Romanticism and Mathematics

Some mathematical developments studied in the Light of ideas of XIX century romanticism.

Summary[1]
Icarus, taken on his flight through the heavens by his waxen wings, turns and falls into the depths of the sea before the terrible power of the Sun; the attribution of metaphysical pretensions to scientific knowledge is yet one more testament to the contradictions which lie uneasily inside man as he realises his frailty in the face of the aspirations of his spirit. Undoubtedly, the contrast that occurs within every man who experiences unlimited desires for a superior or more perfect life, yet at the same time is subject to numerous limitations and the contradictions of the daily routine, is a trial of his need to overcome the physical environment as well as an opening towards transcendental reality, the absolute. Nevertheless, these aspirations and this search have a definite course, and to ascribe an absolutist nature to all human activity or hopes is an error of thinking which diverts it from an objective and true course. Mathematics is knowledge which, due to its

[1] This piece of work was undertaken when the author was as an undergraduate student of mathematics within the framework of the course entitled 'Interdisciplinary Seminar: 19th Century', which was coordinated by the professor María Falk de Losada, of the department of Mathematics and Statistics of the National University of Colombia. The work was presented at the Fourth Symposium of History Philosophy and Mathematics Pedagogy in August of the same year. Today (December 2006), she is a professor in the mathematics Department of the same university.

particular characteristics, is more than prone to such a risk.
From Plato and the Pythagorean School up to that of Bourbaki, the theory of categories or the modern theory of complexity, mathematics has often found itself – to a greater or lesser degree, according to the historical or socio-cultural context – impregnated by this search for 'pure', 'supranatural' or 'innate' ideas that must be attained so as to elevate the discipline above the common condition of man. This necessitates certain personal asceticism without which it is impossible to grasp that 'quasi-divine essence' which mathematical formulas possess. Joined to this position one finds a concept of 'beauty': of 'aesthetics' reflected in the 'purity' of mathematical reasoning, in the 'simplicity' of its forms and in the 'fine sobriety' of its results. All these expressions, often used in mathematical texts and in the approaches of prominent mathematicians [see, for example, that the considerations of Dieudonné (in 5) show a high degree of subjectivity].
Mathematics today is strongly conditioned by its development in 19th century Germany. For this reason it is important to try to understand the social, political and cultural atmosphere of that country at the time. The environment must have necessarily influenced mathematics. It is emphasised here that mathematics is not a procedure but a condition of the spirit, and thus must be related to all other manifestations of contemporary thought.[2] 'A technique or procedure may be invented in isolation but not a spiritual condition,' as Juan Gris, the great exponent of cubism, pointed out.

After trying to find connections with other manifestations of contemporary thought, we have found that there are some fundamental ideas which allow one to suppose an influence of 19th century German romanticism on modern mathematics. The purpose of this work is to demonstrate how some elements present in the sciences can be observed as consequences of an influence of the romantic movement on mathematics, and that these elements are also inherited by the sciences in relation to their degree of 'mathematisation'. The following elements are presented by way of synthesis.

1. The idea of science as absolute knowledge. 'Scientificism.' (Paradoxically, this notion is associated with a pragmatic or instrumentalist concept of science by modern man).
2. The competence of theories not with true postulates but with models of individual occasion, and of the researcher's imagination.
3. A loss of reality. Scorn for the concrete, or what is worse, the declaration of abstractions and self-sufficient models as definitive knowledge of reality
4. Generally, the loss of systems of reference. The controls of mathematic creation are in turn mental creations in which a high degree of subjectivity can be observed.
5. Confusion in what Popper called the limits of rationality: demarcation between science and pseudo science.

This contribution consists of linking these elements to a possible influence of European Romanticism (closely tied to the German idealism of the early 19th century) and to some relevant developments that occurred in mathematics in that century: the so called loss of 'certainty' (emergence of non-Euclidean geometries), and Cantor's theory of transfinite numbers, which are both closely related to the attempts at providing fundaments of mathematics (arithmetic, logic, geometry).

There are three characteristics of romanticism that can be found in mathematics, particularly from the second half of the 19th century. These characteristics are: Hypothesization, Fictionalization, and Disembodiment. This relation between mathematics and Romanticism also involves a close connection with anthropology, sociology and art, as it is unquestionable that Romanticism constitutes a veritable 'injection' given to the European spirit as a whole. These

[2] The article is a textual copy of the one presented in 1993 and the changes which the authoress wishes to incorporate have yet to be made.

relationships are not dealt with fully in this piece of work given their complexity, but constitute a fascinating topic for study in this area.

“Mathematics becomes an expression of the culture of a society at a given moment in history, an explosion of the intellectual concerns of a group, placed within the contradictions and ambivalences of ways of acting and the currents of thinking characteristic of an epoch'. (9) Current mathematics, as has already been hinted, goes on breaking up the demand for truth and transforming it into very conditioned and limited demands (logical processes) of *unity* and coherence of hypothesis (which become converted into axioms for each theory). The fluctuation between degrees of abstraction makes the creation of norms of interpretation (metalanguage) necessary on the part of the 'spectator', who now cannot remain passive. It is here that subjectivity enters and plays an important part: personal interpretations, the complex interior world of the spectator or 'creator', the sense of the 'beautiful', the game, pleasure and that elusive feeling called 'congeniality' constitute the grand irony that mathematics became in the 19th century. It is impossible not to consider with the same eyes North American abstract painting of the 1950's, the resounding impact of Varése, and the theory of categories in mathematics: all these are expressions that only reach their full raison d’être when the spectator deforms and interprets them as he integrates them with his way of seeing the world, and mixes them in the most intimate *fundaments of his being.*

Creativity, individuality and freedom ('The essence of mathematics is freedom,' as Cantor would say), all premises of Romanticism, are worshipped in contemporary mathematics. The glorification of individual perspectives and occasional abstractions, countering concrete and factual experimentation, characteristics of Romanticism, are also characteristics of mathematics. The Romantic sophism that 'hypothesisation and arbitrariness assure freedom and tolerance '(6) converts mathematics into a theory of interpretation.

Another element that manifests a close relationship with Romanticism is the search for *limit* structures in mathematics, as well as their consideration as a medium for maximizing knowledge. The relationship with the notion of *'fundamental type'* that Goethe introduced in modern biology seems to be present here also. Movement and rhythm, perhaps the most beautiful notions that characterise the art influenced by Romanticism, are also found in the mathematics of the late 19th century. “An invigorating parallel lets us see that in mathematics after the revolutionary work of Georg Cantor in set theory, an expansive and contradictory work like that of Mahler, there follows a school of extreme rigor which channels the conquests made by the *displacement of concepts*. Two creators and destroyers of the fixed dimension of traditional forms, Mahler and Cantor, are representatives of new changes of direction that had to be invented to attempt to grasp the structures that in their respective fields reflect the desire for immensity of the last Imperial Europe. Cantor created new entities - their very name is a homage to poetry - transfinite ordinals, upon which construction of the classic binary operations (addition and multiplication) stands, and which allow in some way the measurement of orders of infinity. His work is an attempt to find a principle, which permits the transfer of the most interesting properties of ordinals to abstract sets. Cantor was confronted with the need to *'order well'* any given set. The essential idea aims to achieve a principle for the *encoding* of the world, but the world turns out to be very broad, and the urgency of arriving at a way of *universal individualization* leads to working with entities that are too vast and eventually to the contradictions of set theory. What concerns us here is the desire to explore magnitudes of knowledge until now almost unapproachable: it is the desire to explore very precisely the extensive, broad, charged and diluted forms of the infinite. This was also the period of Monet's water lilies and of the birth of the works of Musil and Joyce. There is a rhythm: the rhythm impregnated with late romanticism transformed into utopia that breaks off, crosses and explodes forms'… 'the mode of logical deduction in present day classical mathematics does not allow (explicit) contradictions and this, seen from outside, seems to be very limiting: a demonstration of congenital rigidity in the possible creations of mathematicians; the testimonies of Poincaré and Hadamard nevertheless give an immense place to the value of intuition, and with them we learn that mathematic creation is often as magical and incomprehensible as any other form of invention; on the other hand, at the moment, logics which admits contradiction, those known as paraconsistents are being explored. Therefore, in mathematics, although a complete comparison with the most abrupt breaks that have happened in music, literature or art cannot be expected, one can try to find a new rhythm on examining the production of the period.'(9)

The absolute confidence that mathematics is accustomed to deposit in certain entities, structures, methods….does not end with the appearance of non-Euclidean geometries. From confidence in certainty and the direct power of conviction of geometric 'evidence' or 'success in applications' the discipline moves on to faith in 'the creative power of pure thought', in the axiomatic method, in the demonstrative method, in logics, in the *recoursive* method… in structuralism, which reminds one of the notion of the 'idea' in the work of Hegel. These are some of the absolutes that each system of mathematics (in a manner of speaking) creates. But the absolute as designed by these systems has a fault: it owes its absolute character to the poetic thinking of the arrogant constructor of systems. With the weakening of his imagination the image of the absolute which he has drawn also fades'. (6)

Finally, and by way of caricature, one may think that 'living dangerously' in mathematics, 'floating on air', is not so bad, as its own raison d'être, its isolation from the outside world does not allow it to know to what degree it approaches or moves away from reality. Mathematics has become an end in itself, and its 'theory of knowledge' affords it an insurance against mishaps that allows the mathematician to pursue his daily routine without any fear. In any case, the profound pleasures, the internal emotions, the imaginary adventures, all in all, the pleasure that the artistic-mathematic daily routine produces is enough to resist the secret and incisive epistemological chisel. But what could happen to the physical and natural sciences in their increasing state of mathematisation? Do they possess a sure epistemology that frees them from possible spasms? Up to what point are they conscious of what occurs within mathematics? What price are they prepared to pay to use a language (sometimes even more than a language) that supposedly offers them a greater degree of precision, of concision, of elegance, of simplicity…?

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* * *

Romanticism in Mathematics

In my book *Filosofia e matematica* [*Philosophy and Mathematics*, Laterza, Rome 2003], on pp. 38-43, I discussed the reason of the change from the concrete view of the axiomatic method to the abstract one at the end of the nineteenth century – a change beautifully illustrated by the Frege-Hilbert controversy. I summarize the main points here, omitting textual evidence and references to save space. I am grateful to Andrea Reichenberger for valuable comments on an earlier version of this text.

It is commonly held that the change from the concrete view of the axiomatic method to the abstract one originated from the creation of non-Euclidean geometries and algebraic theories beyond the traditional theory of equations. Such explanation seems however to be too simple because, for instance, Lobačevskij presented his geometry not as an abstract system but as a concrete investigation of certain physical forces, and Boole presented his algebra not as an abstract system but as a concrete investigation of the operations of the mind by which reasoning is performed. Lobačevskij and Boole did not even present their systems as axiomatic systems.

A more plausible explanation seems to be that the change was due to a somewhat late impact of Romanticism on mathematics.

A basic feature of Romanticism was the claim of a spontaneous and unlimited creativity of the mind and of its power to solve any problem. Romantic philosophers such as Fichte provided philosophical foundations for this claim, and particularly Novalis considered its implications for mathematics.

According to Novalis, humans have an absolute creative power that has no limits, except those they set themselves. This appears in particular from mathematics, for mathematicians create their concepts by an absolutely arbitrary will and determination. The only foundation of their creations is their will. When creating, the mathematician is *en état de Createur absolu*. In a mathematical creation there is just what the creator puts into it.

Mathematical creations are, according to Novalis, creations *ex nihilo* and introduce an abstract world. The abstract character of mathematics derives from the fact that, contrary to the traditional opinion, mathematics has nothing to do with quantity but resembles abstract languages. Both in mathematics and in abstract languages one introduces signs and words arbitrarily. Additionally, in mathematics one introduces principles arbitrarily and deduces all other propositions from them, for to prove something means to deduce it from principles.

Thus for Novalis the method of mathematics is the axiomatic method. Novalis admits that the analytic method is warm, solving, flexible, whereas the synthetic – that is, axiomatic – method is cold, crystallizing, fixing. However he claims that mathematicians use the synthetic rather than the analytic method, for only the synthetic method allows to proceed in an absolutely free way, building pure *a priori* systems without being conditioned by any external stimulus. The synthetic method starts from freedom and ends in freedom, for it introduces concepts and principles arbitrarily and then deduces propositions from them. Moreover it allows mathematicians to solve any problem and to approximate that perfection of will which can be expressed by saying that they can do whatever they want.

The impact of Romanticism on mathematics appears from the assertion − repeatedly occurring at the end of the nineteenth century − that mathematical concepts and principles are absolutely free creations of the human mind.

For instance, Cantor claims that mathematics is completely free in its development, subject only to the requirement of consistency. Dedekind claims that we are a divine race and possess creative power, not merely in material but especially in immaterial things such as mathematics, which is a free creation of the mind. Pieri claims that mathematical concepts are free creations of our mind and axioms are simply acts of our will, so both mathematical concepts and axioms are arbitrary. Poincaré claims that the mathematician deals with facts which he, or, one might even say, his caprice, creates. Vailati claims that mathematics is an arbitrary creation of the mathematician's will, which explains the intimate and basic affinity between mathematics and the creative activity of the artist.

Of course Cantor, Dedekind, Pieri, Poincaré, Vailati had different views on mathematics and different general philosophical outlooks. In particular, some of them were influenced by positivism. But, as regards the latter, today it is widely agreed that several aspects of Romanticism, except the specifically literary ones, occur in positivism, and indeed that the latter can be viewed, in addition to idealism, as one of the two philosophical expressions of Romanticism. Therefore the claim of a completely spontaneous and unlimited creativity of human mind and of its power to solve any problem can be ultimately ascribed to Romanticism.

Such claim found its natural expression in the abstract view of the axiomatic method.

This particularly appears from Hilbert, who maintains that mathematics is absolutely free and its essence consists in its freedom. The latter finds its fullest fulfilment in the abstract view of the axiomatic method, which guarantees the maximum flexibility in research. The concrete view of the axiomatic method allows no freedom and makes one naïvely believe in certain interconnections as though they were dogmas. The abstract view eliminates this naïveté while leaving the advantage of belief. It lets mathematicians be completely free to choose their axioms. Arbitrary axioms, provided that they are consistent, are true, and the things defined by them exist.

According to Hilbert, the abstract view gives us the conviction that the mathematical understanding encounters no limits. Such conviction makes us confident that, by means of the axiomatic method, we will be able to settle every definite mathematical problem, either in the form of an actual answer to the question asked, or by a proof of the impossibility of its solution. This conviction is a powerful incentive to us because it persuades us that we can find a solution of any mathematical problem by pure reason.

Hilbert extended this conviction to the whole science, opposing du Bois-Reymond who claimed that there are insuperable limits in the knowledge of nature and that therefore the scientist cannot simply say *ignoramus* but must, once for all, say *ignorabimus*. According to Hilbert, to Cantor's assertion that the essence of mathematics consists in its freedom, we must add that in mathematics and in natural science there is no *ignorabimus*: We must know, we shall know.

Hilbert's strong opposition to the *ignorabimus* expressed a deep conviction of him, so much so that he even stated an axiom of solvability of all mathematical problems. To stress the importance of such conviction, his motto 'We must know, we shall know' was engraved on his grave-stone. Stating that for the human mind there are no limits in mathematical knowledge and in knowledge generally, such motto is a perfect though somewhat late embodiment of the Romantic Spirit.

As Romanticism in mathematics had a late rise, it had a late end. In 1931 Gödel published his incompleteness results which refuted some of the assumptions on which Romanticism in mathematics depended, in particular the assumption that the method of mathematics is the axiomatic method. That was the end of the Romantic Dream.

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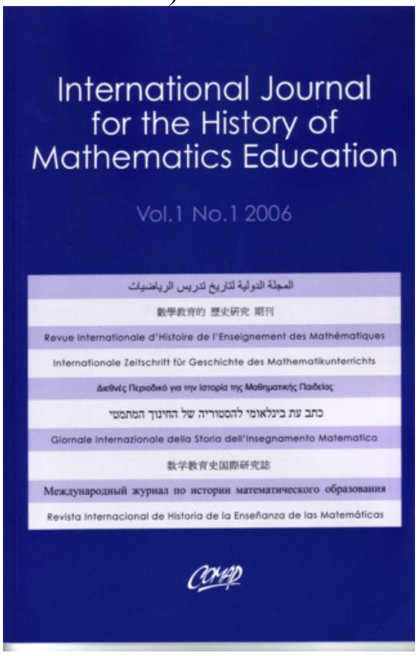
New Journal

Last October, issue no. 1 of the new Journal *International Journal for the History of Mathematics Education*

has been published (editors Gert Schubring, Alexander Karp; publisher: COMAP, Bedford/MA).



The major aim of the *International Journal for the History of Mathematics Education* is to provide mathematics teaching and mathematics education with its memory, in order to reveal the insights achieved in earlier periods (ranging from Ancient time to the late 20th century) and to unravel the fallacies of past events (e.g., reform euphoria). This journal will inform mathematics educators and others about political, social, and cultural constraints (as evidenced by historical events, processes, and periods) in order to improve mathematics instruction. In doing so, the journal aims to overcome disconnected national, cultural, and social histories and to contribute to establishing common themes and characteristic of the development of mathematics instruction in many cultures, differentiating between what constitutes national specificities or particularities and what may be indicative of global trends.
The first issue contains papers by Floriana González and Patricio G. Herbst (USA), Marta Menghini (Italy), Dimitris Patsopoulos and Tasos Patronis (Greece), and João Bosco Pitombeira de Carvalho (Brazil), reviews and information.
The first issue is accessible online, for free, at the address:
http://journals.tc-library.org/index.php/hist_math_ed/
The journal will have two (2) issues each year and will be available both in print and online. At present, issue no. 2 is in preparation for publication. For subscriptions, the address
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can be used.

Gert Schubring

* * *

The HPM Tongxun and the Tongxun Group in Taiwan

Since October of 1998, I began to publish the *HPM Tongxun* on a monthly basis. In fact, we usually merge February and March issues in one issue, and similarly for July and August issues due to Spring and Summer vacations respectively. Therefore, the *Tongxun* has ten issues (both printed and electronic versions) each year and is circulated in printed form of 500 copies to local math teachers and historians of mathematics of the international Chinese community. My article, "The Circulation of the *HPM Tongxun* and Its Relevance to the Mathematics Teacher Community in Taiwan", appeared in the *HPM Newsletter* 50 (July 2002), and aimed to trace the short history of the *Tongxun*. Since then another four years have witnessed the venture taking its shape. By now the *Tongxun* has earned its due reputation among the local community, thanks to more experiments have been conducted by the young HPM activists in order to better serve its original aim; namely, to initiate more activities concerning the HPM as well as the history of mathematics (HM). In this brief report, I would like to add some comments on the articles appeared therein and some involved activities (cf. the list ofcontents of the first 6 issues of volume 9, 2006, which is available from the HPM webpage.)

It should be noted that a special issue on the *Suan Shu Shu* (Han Bamboo Text on Calculation) was published in the November 2000 issue. The article is the first one ever appeared for a comprehensive annotation of the text which was carried out by the editorial board members of the *Tongxun* – all of them being (former) graduate students of mine. This may explain the reason why Joseph Dauben calls my study team on the *Suan Shu Shu* as the Tongxun Group. It is due to the regular publication of special issues on reviewing popular math books released in Taiwan at the December and January Issue. In addition, we also published a special issue on Arabic mathematics (Nos. 11-12, Vol. 4) after the September 11, 2001 tragic event happened in Twin Towers in New York City. Still, another special issue (No. 1, Vol. 8) was devoted to commenting the temporal senior high school math curriculum standards to be put in practices in 2006. On the other hand, we also invite editorial board members serving as guest editors of some special issues. For example, Tsang-Yi Lin was invited as the guest editor to publish a special issue on mathematical induction (Nos. 2/3, 4, Vol. 7); Jung-hong Su to edit a special issue on Heron’s Formula (No. 4, Vol. 9).

As for merits of the publication of the *Tongxun*, a few things should be mentioned here. First, some of them would come to join my research project sponsored by the *National Science Council* (NSC) and *Ministry of Education* (MOE). In this connection, they were invited by their high school colleagues to present reflections on education reform issues. Second, due apparently to our concern with the popular mathematics publications in Chinese version, my team members are invited to write for a column of *Science Monthly*, a major popular science magazine published in Taiwan. The theme of the column is, as I suggest, mathematics in culture. On the other hand, some of the team members have joined me to write a book on *Suan Shu Shu* which has been published in July 2006. To me some of them also add their force in writing collaboratively journal articles on the history of (Chinese or Korean) mathematics. In fact, they are encouraged to present their research papers to the conference on the related topics. For example, several of them were enthusiastic about their participation in the Asia-Pacific HPM 2004 Conference (May 24-28, 2004) held in Taichung. A few of them were invited to present their findings on the history of Chinese mathematics in the Conference on the History of Science on March 26-27, 2005. Some of them have been encouraged to present their comments to the annotation of the *Suan Shu Shu* in the Symposium on the *Suan Shu Shu* that took place in August 23-25 2006. Still, I also encourage my former graduate students, who are teaching at high school, to become local correspondents for the *Tongxun*. This cohesion is able to encourage them promoting the HPM activities once they got a chance to do so – some of them even become local leaders for both the HPM and mathematics teaching. They become very enthusiastic about HPM writing just to share their ideas and vision with their colleagues. As a concluding remark, I would like to mention that a total of about forty correspondents plus ten editorial members can assure the creation of a unique local HPM, in which several of the members act both as a teacher and a historian, despite the fact that a registered learned society is yet to be established, hopefully in the near future.

A list of the contents of of Vol. 9 (1-6) of the *HPM Tongxun* is available from the HPM webpage (go to *Announcements*).

Wann-Sheng Horng,
Taiwan

Announcements of events

Perspectives on Mathematical Practices 2007

March 26-28, 2007
Vrije Universiteit Brussel, Belgium
Confirmed plenary speakers: David Corfield, José Ferreirós, Jens Høyrup, Brendan Larvor, Paolo Mancosu, Yehuda Rav.
Contact Perspectives on Mathematical Practices Conference, c/o Bart Van Kerkhove, Vrije Universiteit Brussel – CLWF, Pleinlaan 2, B-1050 Brussels, Belgium.
e-mail: clwf@vub.ac.be
Visit the PMP2002 website
http://www.vub.ac.be/CLWF/PMP2002

Mephistos

April 6-9, 2007
UCLA, California, USA
http://mephistos.bol.ucla.edu/

5th European Summer University on the History and Epistemology in mathematics education (ESU-5)

July 19-24, 2007

Prague, Czech Republic
For more information,see the HPM Newsletter issues No58, 60 or the ESU-5 website http://www.pedf.cuni.cz/kmdm/esu5

The complete list and abstracts of all activities and the time schedule is on the web site http://class.pedf.cuni.cz/stehlikova/esu5/ProposalsAbsWeb.htm

Important deadlines:

- Early registration by February 28, 2007, registration fee 90 EUR / 50 EUR (for students and Czech school teachers)
- Late registration by May 31, 2007, registration fee 120 EUR / 70 EUR (for students and Czech school teachers)
- Participants wishing to register after May 31, 2007, should pay on the spot 150 EUR / 100 EUR (for students and Czech school teachers)
- For accommodation in student residences, apply via the registration form by March 31, 2007.
- For waived registration fee, apply via the registration form by January 15, 2007.
- Online registration will be closed on May 31, 2007. Participants wishing to register later, should contact nada.stehlikova@pedf.cuni.cz

Mathematics Education In A Global Community *(9th International Conference of The Mathematics Education into the 21st Century Project)*

September 7-12, 2007
Charlotte, North Carolina, USA
See Newsletter 63 for more information.

The First Century of the International Commission on Mathematical Instruction (1908–2008): Reflecting and Shaping the World of Mathematics Education

March 5-8, 2008
Accademia dei Lincei and Istituto dell'Enciclopedia Italiana, Rome, Italy
This symposium in Rome will celebrate the centennial of the International Commission on Mathematical Instruction (ICMI). Starting from a historical analysis of principal themes regarding the activities of the ICMI (reforms in the teaching of the sciences, teacher training, relations with mathematicians and with research, and so on), discussions will focus on identifying future directions of research in mathematics education and possible actions to be taken to improve the level of scientific culture in various countries. The program includes plenary sessions, invited short talks, and working groups. More information can be found at the symposium website:
http://www.unige.ch/math/EnsMath/Rome2008

5th International Colloquium on the Didactics of Mathematics

April 17-19, 2008
Department of Education, Univeristy of Crete, Rethymnon, Crete, Greece
http://www.edc.uoc.gr/5colloquium

ICME-11
July 6-13, 2008
Monterrey, Mexico
http://www.icme11.org.mx/icme11/

HPM 2008
History and Pedagogy of Mathematics
The HPM Satellite Meeting of ICME 11
Plans are being made to call an HPM Meeting to follow ICME-11 due to take place in Monterrey on July 6-13, 2008. Further information will be posted on the HPM website as soon as it becomes available.

Models in Developing Mathematics Education *(10th International Conference of The Mathematics Education into the 21st Century Project)*
September 12-18, 2008
Dresden, Germany
For further information contact
arogerson@inetia.pl

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A note from the Editors

The Newsletter of HPM is primarily a tool for passing on information about forthcoming events, recent activities and publications, and current work and research in the broad field of history and pedagogy of mathematics. The Newsletter also publishes brief articles which they think may be of interest. Contributions from readers are welcome on the understanding that they may be shortened and edited to suit the compass of this publication.

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The views expressed in this Newsletter may not necessarily be those of the HPM Advisory Board.
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65	7 June 2007	1 July 2007
66	12 October 2007	1 November 2007
67	12 February 2008	1 March 2008

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