

International Study Group on the Relations Between
the HISTORY and PEDAGOGY of MATHEMATICS
An Affiliate of the International Commission on
Mathematical Instruction

No. 57 November 2004

This and earlier issues of the Newsletter can be downloaded from our website
<http://www.mathedu-jp.org/hpm/index.htm>

Message from the Chair

It is a great honour for me to have been proposed as the HPM chair for the next four years, and I would like to thank all colleagues, who thought that I could be useful in this position. At the same time, however, I feel a great responsibility for that and I hope that I will be able to contribute in making this International Study Group's activities flourish, as much as possible.

I was born in Iraklion, in the island of Crete, Greece and I have studied Mathematics at Athens University, Greece, Astronomy at Sussex University, UK and obtained a doctoral degree in Theoretical Physics from the Université Libre de Bruxelles, Belgium. In the last 15 years I have been teaching Mathematics and Physics at the Department of Education of the University of Crete. My research interests include areas both of Theoretical Physics and of Mathematics and Physics Education. Thus, although I have been educated as a mathematician and as a theoretical physicist, I have been involved in Mathematics Education for several years and have always had a strong interest in the history of Mathematics and Physics, their mutual interrelation and the didactical implications of their history.

I have been involved in the HPM activities since 1993 and I very soon realized that these activities are based on international cooperation among the members of the group.



C. Tzanakis, the new Chair

I do believe that all of us consider the HPM group to be a group of people with overlapping scientific interests and with a *common* objective to contribute to humanizing Mathematics, revealing its cultural and multidimensional nature and improving Mathematics Education through the didactical implications of the historical development of Mathematics. This point of view has been stressed on several occasions, the latest one coming to my mind being in the lively discussion during the very last HPM session at *ICMI-10* in Copenhagen, last July. Therefore, in my view, being the Chair of HPM means nothing more and nothing less than acting as a kind of motive force for coordinating initiatives, collaborating on the

realization of suggestions and collecting relevant information. To this end, I have in mind

- 1 to collaborate closely with the former Chairs of the group and all members of the Advisory Board, whom I thank for their willingness to help. In this way, I hope that I will have a better overview of activities in progress, or ideas and initiatives to be realized in practice.
- 2 to assist the regular publication and continuous enrichment of the *HPM Newsletter* and the constant improvement and updating of the HPM website. Both the *Newsletter* and the *website* can be important tools for making easier the contact among the members of the group and for increasing its visibility.
- 3 to actively support magazines or journals related to the HPM perspective and encourage or motivate the publication of special issues of other journals devoted to themes that emphasize the historical dimension in Mathematics education.
- 4 to collaborate in organizing local or international activities (meetings, conferences, colloquia etc) that will provide the opportunity to bring together people, who are interested in integrating history into Mathematics Education. Such activities will hopefully stimulate new collaborations and foster further initiatives that will bring closer mathematicians, historians of Mathematics and mathematics educators, who are eager to contribute to the improvement of Mathematics Education around the world.

Of course, all this will remain empty words, unless there are people who are willing to collaborate in getting as many people as possible involved in and get informed about the HPM activities.

Therefore, I would like to invite each and

every member of this International Study Group, to make suggestions and take initiatives that will contribute to the realization of the HPM perspective. My desire and duty is to support all of them.

Constantinos Tzanakis, Chair

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Interview with the former HPM Chair, Fulvia Furinghetti

by C. Tzanakis

Dear Fulvia,

You have been the chair of the HPM group for the period 2000-2004. Thanks to your tireless efforts, Peter Ransom's wonderful editorship of the *HPM Newsletter* and the support from the members of the HPM Advisory Board, the HPM activities have flourished in the past four years. In this way, the relation between the History of Mathematics and Mathematics Education has expanded even more into an interesting, active and fruitful domain of research, in which mathematicians, historians and mathematics educators meet in a constructive way.

In an effort to have a deeper awareness of what has been achieved and how we could continue to contribute to humanizing Mathematics, revealing its cultural and multidimensional nature and improving Mathematics Education through the didactical implications of the historical development of Mathematics, let me ask a few questions, motivated from your first Message from the Chair (Newsletter no 44, November 2000, p.1) and your own experience as the previous HPM Chair:

C. Tzanakis: *In your original plan, you aimed at "... [keeping] in contact those interested in the field" It would greatly help all of us, to draw from your experience and have some ideas of how this has been, or can be accomplished.*

F. Furinghetti: I like to think that my goals have been achieved. Obviously, there have been moments in which I felt that things should have been better managed by me. In

reflecting on my way of acting in the past four years I realize that only now, at the end of my chairing period, I have the suitable experience to be the HPM chair. The HPM Study Group has a fuzzy structure; no official registration when entering the Group, no annual fees, no precise list of members (except of the Advisory Board). The only way to keep the identity of the Group is to stress the cultural background and aims of the members. For this reason regular contacts are so important.

Firstly, I tried to take care of personal contacts. I think that I may claim that not a single message concerning the relationship between history and pedagogy sent to me remained without answer. Colleagues, young researchers, doctoral students, teachers who asked me for references, papers, advise had my answer (in the limit of my competencies and possibilities). In addition, I tried to accept all the invitations as the HPM chair.



F. Furinghetti (with F. Fasanelli)

At a more official level, I tried to revitalise contacts, first of all, to have the *Newsletter* published again. It was John Fauvel's choice to have Peter Ransom as the editor, and the past four years in which 12 issues of the *Newsletter* have been published make evident that it has been a great choice. Peter has acted very well, although I feel that the members of the HPM have not always been as collaborative as we (Peter and I) were expecting. The set of regular contributors to sections such as "Have you read this?" or "Reviews" was very small. Often announcements and reports of regional HPM meetings were not sent.

After the rebirth of the *Newsletter* the second enterprise was to have a website. Initially we were hosted in Americas-HPM website, until Masami Isoda volunteered to be our webmaster.

The third keep-in-contact initiative was the creation of the logo. This is something less important than to have the newsletter and the website, but it has also some meaning for stressing the group's identity.

C. Tzanakis: *You have also been thinking of "... 'promoting' the interest in the field in the countries in which the tradition is less alive". This is a really difficult task. Do you think that some progress has occurred in this direction?*

F. Furinghetti: My intention was not to convince people that to use history is the best way to teach mathematics; rather I tried to make people aware that history is a possibility, with advantages and disadvantages as it happens in the case of other teaching means. This awareness may support the use of history, or simply to foster the introduction of the history of mathematics in teachers' education. The choice of Uppsala as the location for the 2004 HPM Satellite Meeting was done for various reasons; one reason was to encourage the orientation of local people towards history.

C. Tzanakis: *As it can be readily seen from your final report (Newsletter No 56, pp.1-5), you have tried hard and successfully "... to continue the tradition of the meetings (regional or international) focused on issues related to the HPM perspective". Do you think that this tradition can be further developed and in which way? For instance, by encouraging activities of a more local character and interest, or by coordinating and realising research programs of wide influence like the recent ICMI Study on the Role of the History of Mathematics in the teaching and learning of Mathematics?*

F. Furinghetti: Meetings (regional or international) focused on issues related to the HPM perspective are successfully held when there is a convenient background. The convenient background is created by a net of studies and publications, which inform researchers and by local initiatives involving

teachers as well. For me, the best is to have sessions related to the HPM perspective in meetings not specialized to the HPM. A good example may be the conference held in Marrakech in 2002, where there have been talks on the use of history, together with talks by historians. Or, the conference on math education in 2002 in Crete (ICTM2), where there were talks about the use of history and also a panel on this subject. Due to the nature of the group, which involves different competencies, it is good to offer occasions to combine these competencies and to have opinion exchanges. In 2005, CERME 4 will be held in Spain. I am glad to read in the program that a panel will be titled "History and Theory of Mathematics Education". Of course, we need also more specialized conferences, such as the Satellite Meetings of HPM at ICME's.

C. Tzanakis: *As you have clearly stated in your original plan, it would be extremely important if the HPM perspective were a reference point for the curriculum developers and teacher educators. Do you think that in the first years of this millennium, there is some progress in improving and deepening teacher educators' and curriculum developers' awareness of the role of History in Mathematics Education? - for instance, through the ICMI Study on the Role of the History of Mathematics in the teaching and learning of Mathematics. In particular, do you think that this study and its outcome (the ICMI Study volume 6, "History in Mathematics Education: The ICMI Study") have helped to develop cooperation among researchers, teachers and mathematics educators, to think over again on basic issues of teaching and learning Mathematics, and to influence education in practice?*

F. Furinghetti: I am very much concerned about teachers' education. I plan to talk about that in relation with history in the forthcoming conference in Crete (in April 2005). I think that teachers are the chief characters in the school drama, even more than students. There are some (a few) studies on the role of history in changing their attitude towards mathematics and its teaching. Teachers' reaction to stimuli towards history depends on

their background. As I have written elsewhere, the use of history is not for all, but only for very motivated teachers in convenient circumstances. As far as Italy is concerned, we have succeeded to get the permission from the Ministry of Education that courses for in-service teachers training are partly devoted to history. For instance, there has been a course on algebra, in which the time was divided into four equal parts. One part was devoted to educational problems in the teaching of algebra, one to new contents of algebra, one to technology and one to history.

As far as the ICMI Study book is concerned, it is difficult to say about its impact. Again referring to Italy I may say that teachers read English texts with difficulty, and this is even truer for curriculum developers, who are our inspectors.

C. Tzanakis: *Related to the above, is the more general question, whether in the context of the HPM group, it would be possible to develop an agenda of research, in the sense of pointing out basic issues and lines of approach in this area.*

F. Furinghetti: I think we have to care of having young researchers, doctoral students working in the field of HPM. It's been a few years now that a summer school for young researchers in mathematics education has been held in different places of Europe. I would like that in this school history has a place as technology and other issues have. Another issue that I would like to stress is the development of empirical research and analysis of experiments carried out in the classroom.

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Are you missing anything?

Please send articles, notices, information on meetings and so on to the editors.

Interview with the former HPM Editor, Peter Ransom

by the new co-editors

New Editors: *Dear Peter, since November 2000, when you took over as editor, the shape and the function of the Newsletter has become more structured. It has also become much bigger. It is clear that the changes were largely due to your own work and dedication, but you must also have had the help and support of many others. How did you manage to get all this help and support?*

P. Ransom: All I did was to write to all the distributors to let them know I was the new editor, and to let them know when the Newsletter was to be sent to them regularly. Then about 6 weeks before the next issue I sent a letter (but this was replaced by email message to all but two distributors) reminding them that if they had anything to include would they please send it to me two weeks before the Newsletter was to be sent. I also sent it to the Chair (Fulvia Furinghetti) and former Chair (Jan van Maanen). Fulvia sent me her Chair's remarks and got other people to send me a few things (like obituaries and conference reports).

My school agreed to fund the postage of sending out the paper copy to all distributors. All the rest was done by me: - the type setting, the typing up of handwritten articles (by Professor R. C. Gupta of India), emailing, photocopying, envelope addressing and stuffing. I also copied details of meetings from the BSHM website.

New Editors: *Which are the main themes that you thought important for the Newsletter?*

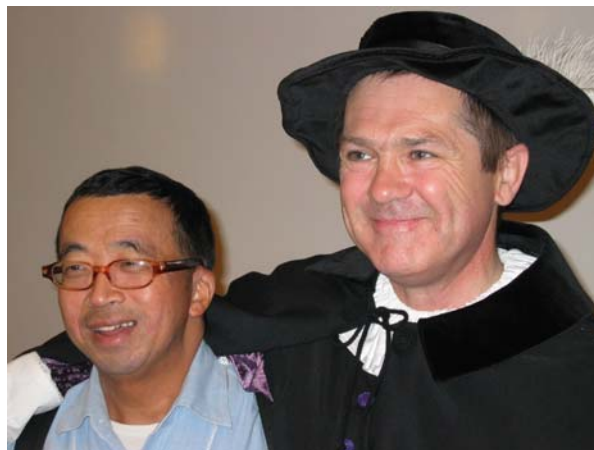
P. Ransom: I considered it most important to include as many images as possible. To me pages of text do not encourage people to read the Newsletter.

I think the best themes were the conference reports (especially if there was a picture), articles and notice of events. I feel it important to encourage others to write about their work and interests, which is why Professor Gupta was invaluable as his articles

were quite varied.

The section "*Did you read?*" I felt was also important, but unfortunately sometimes there was not many contributions.

New Editors: *Did you find that the important role of the Newsletter as a vehicle for providing information about activities and events dominated to the extent that it inhibited the opportunity to develop other areas?*



P. Ransom (with M-K. Siu)

P. Ransom: No. Everything that was submitted was published. I decided that the number of pages did not matter, just that it was a multiple of 4 so nobody felt there was wasted space. If I had space left I tried to start on a new page so that the space was fairly evenly distributed.

New Editors: *What, in your opinion, are the important things for the new co-editors to pay attention to? Are there any areas you think particularly important to develop?*

P. Ransom: It is important to mention anybody's research so that others can make contact if they so wish. Also to produce it at regular intervals so that people get used to expecting it at particular times (though you will be dependent on the distributors).

New Editors: *What is the importance of HPM to you?*

P. Ransom: I enjoyed the communication with people all over the world with a common interest. To me I take great pride in managing to meet deadlines with as professional a product as possible. It was a great feeling of belonging and community at the HPM Uppsala meeting.

New Editors: *In which direction would you wish HPM to head in the future?*

P. Ransom: I would like to see more young people getting involved, which is why I think it absolutely fantastic that there is a good cross-section in the editorship. It would be great to get young researchers using HPM to request information about their work.

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Work in progress

We encourage young researchers in fields related to HPM to send us a brief description of their work in progress or a brief description of their dissertation.

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Reviews

Good News from the Americas' Section of HPM

Historical Modules for the Teaching and Learning of Secondary Mathematics

Editors: Victor J. Katz and Karen Dee Michalowicz. Published by the MAA.

Producing historical materials for use in the mathematics classroom has always been an activity, which is a main characteristic difference of the HPM group from the rest of the mathematical world (i.e. mathematicians, historians of mathematics, mathematics educators, etc.). A small sample of a few available in English and well-known documents is the following (in chronological order):

- NCTM's 31st Yearbook on *Historical Topics for the Mathematics Classroom* (1969/1989),
- A. Arcavi & M. Bruckheimer's source-work collections for in-service and pre-service teacher courses on *Negative Numbers, Irrational Numbers and Equations* (1983-85),
- IREM's *Papers Towards a Historical Perspective in the Teaching of Mathematics* (1988) and *Stories of Problems* (1992),

- R. Laubenbacher & D. Pengelley's *Graduate Courses Based on Original Historical Sources* (1992 - present).

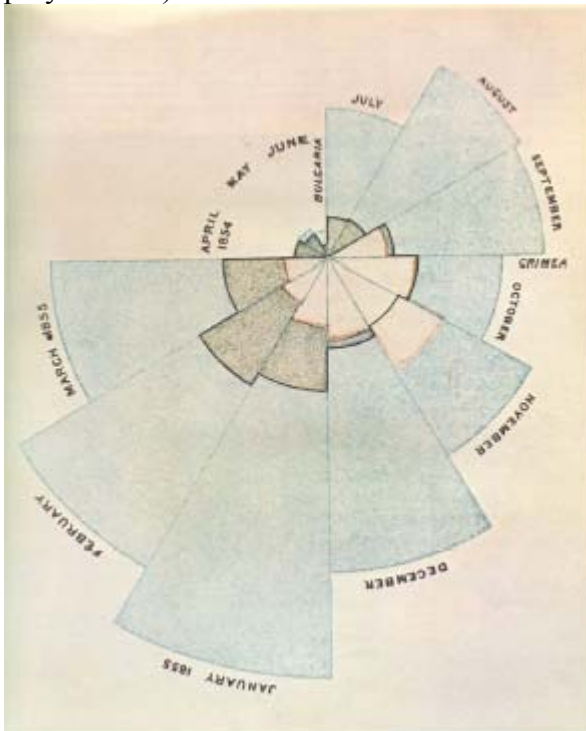
Recent developments in mathematics education, like those exposed in the NCTM's *Curriculum and Evaluation Standards for School Mathematics* (1989) or *Principles and Standards for School Mathematics* (2000), have brought to the fore new challenges and opportunities for using history in the teaching of mathematics in U.S.A.. Fifteen years ago, Ch. V. Jones, one of the founders of HPM, set out his concern on this point in the following words:

... to use history in a fashion suggested in the *Standards* requires developing historical modules, writing a different kind of history of mathematics book, and utilizing history in textbooks at a more profound level than biographical or historical vignettes. These are all activities that the HPM in the United States and Canada, in particular, has promoted – although not with very remarkable results. With the publication and adoption of the *Standards* I would hope that we would see – at long last in America – more writing of historical materials suitable for use directly in the mathematics classroom in the spirit of the *Standards*. It is clearly the time for the Americas Section of HPM to take a leading role in the American education reform movement. (Jones, 1990)

Sharing the same concern, a group of people well-known to the readers of this Newsletter, among others F. Fasanelli, K. D. Michalowicz, V. Katz and F. Rickey, started to work in this direction ten years ago. A major step in attaining this goal has been the establishment of the *Institute in the History of Mathematics and its Use in Teaching* (IHMT), administered by the Mathematical Association of America and funded by the National Science Foundation. In IHMT, people coming from U.S. high schools, colleges and universities have had an intense summer program in the history of mathematics and its use in teaching, since 1995. A special part of the IHMT has been

the Historical Modules Project, co-directed by V. Katz and K.D. Michalowicz, where about thirty high school and college teachers of mathematics teamed up to produce the following modules:

1. *Archimedes* (activities from the work of Archimedes)
2. *Combinatorics* (the elementary formulas for combinations and permutations along with an introduction to probability)
3. *Exponentials and Logarithms* (the development of the exponential and logarithmic functions with applications)
4. *Functions* (the general idea of a function, with illustrations from many sources)
5. *Geometric Proof* (an historical study with numerous examples)
6. *Lengths, Areas and Volumes* (activities from around the world dealing with the measurement of these quantities)
7. *Linear Equations* (the idea of a proportion along with the solution of linear equations and systems of linear equations)
8. *Negative Numbers* (how these quantities are used and why)
9. *Polynomials* (methods for solving quadratic and cubic equations, as well as more general polynomials)



A chart by Nightingale in the Statistics Module

10. *Statistics* (basic concepts of statistical reasoning, including graphs)

11. *Trigonometry* (from the creation of a sine table to the measurement of plane and spherical triangles)

Last July, during the 10th *International Congress on Mathematics Education (ICME 10)* in Copenhagen, Denmark and its *Satellite Meeting of HPM (HPM 2004)* in Uppsala, Sweden, this more than 1300 pages huge resource material has become available in CD-form to the international audience.

A most distinctive feature of these modules is that both their structure and content form an almost complete course in secondary education mathematics, inspired from and based on the history of mathematics. In V. Katz and R. Kessler's words:

Each of the lessons in our module was designed to actively involve the student. These activities were not designed as history lessons with a mathematical flavour, but rather mathematics with an historical perspective. The various lessons were created as independent choices. It is possible to choose a single part of the module and use it at an appropriate time during the year. The activities were designed to promote interest and understanding, often incorporating the use of technology. (Katz & Kessler, 2000)

Each historical module is structured according to the following plan:

1. Introduction
(With: Historical Overview, Rationale, Activity Summaries, Suggested Courses for Using Module Activities.)
2. Activities, consisting of
 - 2a. Teachers' Notes
(With: Appropriate Level, Materials, Time Frame, Objective, When to Use, How to Use, Answers and Solutions.)
 - 2b. Students' Pages
(With: Worksheets, Questions for Discussion, Exercises, Plays, and Dialogues)
3. Illustrations and Transparencies
4. Suggestions for Student Projects
5. Bibliography



A Reisch woodcut in the Introduction

It is my firm opinion that the launching of the *Historical Modules* is not only good news for enthusiastic teachers, having an urge to use history in their mathematics classroom, but also provides grist for the mill for the rest of the mathematical world. Several issues of great concern for mathematics educators, or historians of mathematics can be found in the historical overviews, or the activities contained in the modules. As an

exemplification, let me cite the following:
 (a) In the historical overview and the activities on piecewise functions, contained in the *Functions*-module (pp. 9ff, 43ff) one can find an excellent introduction to the mathematicians' changing views on central notions like "continuity" of a function. At a time when the concept of function was identified with an "algebraic formula" expressing some physical law (a common view of many students today), mathematicians, like L. Euler, or J. L. Lagrange considered every piecewise function (consisting, say, of two formulas and so expressing two different laws) to be discontinuous. After changing the standards of rigour in *Analysis*, A. L. Cauchy showed that this concept of continuity is not only unsatisfactory from a mathematical point of view, but even a contradictory one.

This is a lesson from history, focussing on the crucial role played by epistemological obstacles and conceptual changes in the evolution of mathematical knowledge, and so very well suited to the teaching and learning of mathematics.

(b) On the other hand, in the activity named "Geometric Algebra and Quadratic Equations – Solving quadratics through geometry", of the *Geometric Proof*-module (pp. 83 – 84), students are guided to solve the equation $x^2 + 21 = 10x$ through an application of a geometric proposition of the 2nd book of Euclid's *Elements*. Given that the very form of this equation and the process of algebraic factorisation leading to the quadratic formula, are by-products of Descartes' revolutionary step to apply symbolic algebra in geometry, this activity seems to turn history upside-down. Activities of this kind raise some much-debated issues in the historiography of mathematics, which have recently entered the field of HPM as critical voices (Fried, 2001; Nooney, 2002).

All the above suggest that the emergence of the *Historical Modules* is a mark of the vigorousness of our field, which greatly facilitates teaching mathematics in a historical perspective and promotes further discussion on the benefits and limits of this endeavour. The people of the Americas Section of HPM have now the difficult task to diffuse the use of the *Historical Modules* to a wider circle of teachers in USA (or elsewhere), beyond those involved in IHMT and the Project. The diffusion and gathering of relevant data will certainly help to shed new light on certain fundamental and recurring questions, like those posed recently by J. van Maanen:

- How can we be sure that the history of mathematics helps in the teaching and learning of mathematics?
- And, is this universally true, or does history help only in certain learning environments, with students and teachers at a certain level or with certain mathematical subjects?
- And, if history helps, what then is the rationale behind it? (Maanen, 2002)

References

- Charles Jones (1990), Reform in American Education. *Newsletter of the International Study Group on the Relations between History and Pedagogy of Mathematics (HPM)* No 19, pp. 8-10.
- Victor Katz & Rebecca Kessler (2000), Historical Modules Project. *Mathematics Teacher*, Vol. 93, No 8, p. 728.
- Michael Fried (2001), Can Mathematics Education and History of Mathematics Coexist? *Science & Education* 10, pp. 391-408.
- Kevin Nooney (2002), A Critical Question: Why Can't Mathematics Education and History of Mathematics Coexist? *The Mathematics Educator* Vol.12, No1, pp. 1-5.
- Jan van Maanen (2002), Research on History in Mathematics Education in the Netherlands: The 'Reinvention Studies'. In: Fou-Lai Lin [ed.], *Common Sense in Mathematics Education*, pp.191-201. Taipei, Taiwan: National Taiwan Normal University.

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In you would like to be involved in reviewing books or magazines for this section, please send your contact details and area(s) of interest to the editors who will forward books or magazines for review as and when they become available.

The views expressed in this section are the views of the reviewers and may not necessarily be those of the HPM Advisory Board.

If you wish for a book to be reviewed, please send it to the editors who will arrange for it to be reviewed.

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Ph.D. theses

Eliana da Silva Souza:

A prática social do cálculo escrito na formação de professores: A história como possibilidade de pensar questões do presente. (UNICAMP, Campinas, Aug. 2004)

Thesis director: A. Miguel; Jury: J.M.L. de Matos, M.A. Miorim, G. Schubring, A.C. Carrera de Souza, A.R. Lanner de Moura

The work presents a novel approach to research on the use of mathematics history in teaching mathematics, combining experimental sessions and historical research. The researcher conducted experimental sessions with experienced primary school teachers confronting their practice in teaching the written algorithms of the arithmetic operations with knowledge obtained from her research on the diversity and differences of these procedures, as evidenced by the first printed arithmetic textbooks of the 15th and 16th centuries. This distancing made the teachers aware of the fact that what they had assumed to be natural and self-going procedures are rather the result of social and cultural choices and conventions. A new awareness for the historical nature of mathematical knowledge emerged, which is relevant for their teaching practice. A publication of the thesis is in preparation.

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Have you read these?

Antónia, Rosa: Integrating History of Mathematics into the Mathematics Classroom, *Centro de Matematica da Universidade do Porto, Preprint*, 2001-25, <http://www2.fc.up.pt/cmup/v2/frames/publications.htm>

Bernard, Alain: Ancient Rhetoric and Greek Mathematics: A Response to a Modern Historiographical Dilemma, *Science in Context*, 16(3), 2003, pp. 391-412.

Boyé, Anne: Quelques Éléments d'Histoire des Nombres Négatifs, in *González, Miguel Hernández (ed.) : The Role of the History of Sciences in Secondary Education*, [Projecto Penélope-European Project Socrates (Comenius 3.1) TF-168/2002], La Orotava, 2002, pp. 127-145.

Boyé, Anne: Les Femmes dans les Mathématiques, in *González, Miguel Hernández (ed.): The Role of the History of Sciences in Secondary Education*, [Proyecto Penélope-European Project Socrates (Comenius 3.1) TF-168/2002], La Orotava, 2002, pp. 206-213.

Chemla, Karine: What Was a Mathematical Problem in Ancient China? *Conference: Disunity of Chinese Science*, University of Chicago, May 10-12, 2002, <http://uts.cc.utexas.edu/~rhart/conferences/chinesescience/>

Chemla, Karine: Generality above Abstraction: The General Expressed in Terms of the Paradigmatic in Mathematics in Ancient China, *Science in Context*, 16(3), 2003, pp. 413-458.

Christianides, Giannes: *Topics on the History of Mathematics. Egyptian, Babylonian and Greek Mathematics. History and Philosophy of Science* [in Greek], Heraklion: Panepistemiakes Ekdoseis Cretes, 2003.

Corry, Leo: Introduction: The History of Modern Mathematics – Writing and Rewriting, *Science in Context*, 17(1-2), 2004, pp. 1-21.

Dahan Dalmedico, Amy: Early Developments of Nonlinear Science in Soviet Russia: The Andronov School at Gor'kiy, *Science in Context*, 17(1-2), 2004, pp. 235-265.

Delve, Janet: The College of Preceptors and the *Educational Times*: Changes for British mathematics education in the mid-nineteenth century, *Historia Mathematica*, 30, 2003, pp. 140-172.

D'Enfert, Renaud : *L'enseignement mathématique à l'école primaire de la Révolution à nos jours. Textes officiels. Tome 1: 1791–1914*, Paris, Service d'histoire de l'éducation, Institut national de recherche pédagogique, 2003.

Dennis, David: The Role of Historical Studies in Mathematics and Science Educational Research, in *Dick Lesh & Anthony Kelly (Eds.), Research Design in Mathematics and Science Education*, Mahwah, NJ: Lawrence Erlbaum, 2000, http://www.oswego.edu/multi-campus-nsf/David_Dennis_Chapter.htm

Epple, Moritz: Knot Invariants in Vienna and Princeton during the 1920s: Epistemic Configurations of Mathematical Research, *Science in Context*, 17(1-2), 2004, pp. 131-164.

Fauvel, John: The role of history of mathematics within a university mathematics curriculum for the 21st century, *Teaching and Learning Undergraduate Mathematics Newsletter*, No 12, 2000, <http://www.bham.ac.uk/ctimath/talum/newsletter/talum12.htm#fauvel>

Ferreirós, José: The Motives behind Cantor's Set Theory – Physical, Biological, and Philosophical Questions, *Science in Context*, 17(1-2), 2004, pp. 49-83.

Filep, L.: The Development, and the Developing of, the Concept of a Fraction, *International Journal for Mathematics Teaching and Learning*, 2001, <http://www.ex.ac.uk/cimt/ijmtl/lffract.pdf>

Fried, M.N. and Unguru, S.: Apollonius of Perga's Conica: Text, context, subtext. Leiden: Brill, 2001.

Georgiadou, M.: *Constantin Carathéodory. Mathematics and Politics in Turbulent Times*, Springer-Verlag, 2004.

Gerdes, Paulus: Basketry, Geometry, and Symmetry in Africa and the Americas, *Visual Mathematics* (Special E-Book issue) <http://www.mi.sanu.ac.yu/vismath/gerdbook/gerdcontents/contents.htm>



Illustration from Gerdes.

Giacardi, Livia: *Educare alla scoperta*. La lezioni di Carrado Serge alla Scuola di Magistero, *Bollettino della Unione Matematica Italiana*, (8), 6-A, 2003, pp. 141-164.

Giacardi, Livia: I Manuali per l'Insegnamento della Geometria Elementare in Italia Fra Otto e Novecento, in *Chiosso, Giorgio (ed.): TESEO Tipografi e editori scolastico-educativi dell'Ottocento*, Editrice Bibliografica, 2004, pp. xcvi-cxxiii.

Gray, Jeremy J.: Anxiety and Abstraction in Nineteenth-Century Mathematics, *Science in Context*, 17(1-2), 2004, pp. 23-47.

Imhausen, Annette: Egyptian Mathematical Texts and Their Contexts, *Science in Context*, 16(3), 2003, pp. 367-389.

Katz, Shaul: Berlin Roots – Zionist Incarnation: The Ethos of Pure Mathematics and the Beginnings of the Einstein Institute of Mathematics at the Hebrew University of Jerusalem, *Science in Context*, 17(1-2), 2004, pp. 199-234.

Lefort, Xavier: Rapport sur l'enseignement de l'histoire des sciences en France: L'expérience des IREM, in *González, Miguel Hernández (ed.): The Role of the History of Sciences in Secondary Education*, [Proyecto Penélope-European Project Socrates (Comenius 3.1) TF-168/2002], La Orotava, 2002, pp. 123-126.

Lefort, Xavier: Histoire des Longarithmes un Exemple du Développement d'un Concept en Mathématiques, in *González, Miguel Hernández (ed.): The Role of the History of Sciences in Secondary Education*, [Proyecto Penélope-European Project Socrates (Comenius 3.1) TF-168/2002], La Orotava, 2002, pp. 142-151.

Mendez, Edith Prentice: A History of Mathematics Dialogue in Textbooks and Classrooms, *The Mathematics Teacher*, 94 (3), 2001, pp. 170-173.

Nooney, Kevin: A Critical Question: Why Can't Mathematics Education and History of Mathematics Coexist?, *The Mathematics Educator*, 12 (1), 2002, <http://jwilson.coe.uga.edu/DEPT/TME/issues/v12n1/6Nooney.pdf>

Matos, J.: The Historical Development of the Concept of Angle (1), *The Mathematics Educator*, 1 (1), 1990, pp. 4-11, <http://jwilson.coe.uga.edu/DEPT/TME/issues/v01n1/2matos.pdf>

Matos, J.: The Historical Development of the Concept of Angle (2), *The Mathematics Educator*, 2 (1), 1991, pp. 18-24, <http://jwilson.coe.uga.edu/DEPT/TME/issues/v02n1/4matos.pdf>

Miguel, Antonio and Maria Ângela Miorim: *História na Educação Matemática. Propostas e desafios* Editora Autêntica, Belo Horizonte/MG (Brazil), 2004. ISBN 85-7526-120-7

This book presents a complement to the by now classical ICMI Study History in Mathematics Education (2000). Its authors, researchers at the interface of history and philosophy of mathematics with mathematics education, have realized a thorough investigation of the theoretical foundations underlying the various present approaches to the use of mathematics history in mathematics education. Their analysis and critique refers to the social, psychological, epistemological, and historical dimensions of these approaches. The book is a "must" for all those

interested in the use of mathematics history in teaching mathematics.

Outside Brazil, the book can be ordered via vendas@autenticaeditora.com.br

Mueller, Ian: Remarks on Euclid's *Elements* I,32 and the Parallel Postulate, *Science in Context*, 16(3), 2003, pp. 287-297.

Netz, Reviel: Introduction: The History of Early Mathematics – Ways of Re-Writing, *Science in Context*, 16(3), 2003, pp. 275-286.

Netz, Reviel: *The Transformation of Mathematics in the Early Mediterranean World*, Cambridge University Press, 2004, 208 pp.

Ponte, J.P.: The History of the Concept of Function and Some Educational Implications, *The Mathematics Educator*, 3 (2), 1993, <http://jwilson.coe.uga.edu/DEPT/TME/issues/v03n2/ponte.html>

Romano, Antonella: Réflexions sur la construction d'un champ disciplinaire : les mathématiques dans l'institution jésuite à la Renaissance, *Paedagogica Historica*, 40(3), 2004, pp. 245 – 259.

Ross, S./ Pratt-Cotter, M.: Subtraction in the United States: An Historical Perspective, Matos, J.: The Historical Development of the Concept of Angle (1), *The Mathematics Educator*, 10 (2), 2000, pp. 49-56, <http://jwilson.coe.uga.edu/DEPT/TME/issues/v10n2/5ross.pdf>

Rowe, David E.: Making Mathematics in an Oral Culture: Göttingen in the Era of Klein and Hilbert, *Science in Context*, 17(1-2), 2004, pp. 85-129.

Ruso, L.: *The Forgotten Revolution. How Science Was Born in 300 BC and Why it Had to Be Reborn*, Springer-Verlag, 2004.

Saito, Ken: Phantom Theories of pre-Eudoxean Proportion, *Science in Context*, 16(3), 2003, pp. 331-347.

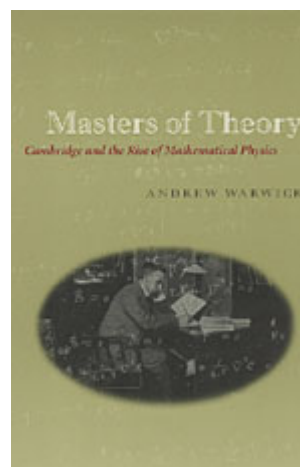
Scholz, Erhard: Hermann Weyl's Analysis of the "Problem of Space" and the Origin of Gauge Structures, *Science in Context*, 17(1-2), 2004, pp. 165-197.

Sidera-Lytra, P./Sideras, A./Stamou, G.: *Constantin Carathéodory. 50 letters to his colleagues in Goettingen* [in Greek], Thessaloniki, Aristotle University of Thessaloniki, 2004.

Taisbak, Christian Marinus: Exceeding and Falling Short: Elliptical and Hyperbolic Application of Areas, *Science in Context*, 16(3), 2003, pp. 299-318.

Tsimpourakis, Dimitris: *The Trigonometry in Ancient Greece* [in Greek], Athens, Atrapos Publication, 2004.

Warwick, Andrew: *Masters of Theory. Cambridge and the Rise of Mathematical Physics*, Chicago and London, The University of Chicago Press, 2003.



* * *

Have you been here?

The HPM website is at

www.mathedu-jp.org/hpm/index.htm

The British Society for the History of Mathematics web site at

www.dcs.warwick.ac.uk/bshm/

has many links to related sites.

The Italian Society of History of Mathematics web site is

www.dm.unito.it/sism/index.html

The HPM-Americas web site is

www.hpm-americas.org

The latest (number 29) AMUCHMA newsletter on the history of mathematics in Africa can be found at

www.math.buffalo.edu/mad/AMU/amuchma_online.html

All the earlier issues are available on the same web page.

For a history of HPM visit

http://mcs.open.ac.uk/puremaths/pmd_department/pmd_fauvel/HPM_%20history.htm

History and Epistemology for the Teaching of Mathematics has been activated at the address:

www.syllogismos.it

On the site it is possible to find material relating to the teaching of mathematics and some historical references which will be useful in the field of mathematics. Every/any suggestion to improve such a site conceived mainly in terms of helping colleagues involved in education and in particular in teaching will also be welcomed.

Make sure you visit Iris Gulikers' website. She has produced a unit for schools so they can replicate some of the techniques used by surveyors and this can be found on her website, some of which is in English as well as Dutch.

<http://members.home.nl/gulikgulikers/WiskundePagina.htm>

Convergence-where mathematics, history and teaching interact, [A Magazine of the Mathematical Association of America]
<http://convergence.mathdl.org/convergence/1/>

La Matematica in Italia (1800-1950), [Il Giardino di Archimede,
Un museo per la matematica]
<http://www.math.unifi.it/matematicaitaliana/>

Topic Study Group (TSG) 17: The role of the history of mathematics in mathematics education, [ICME-10, Copenhagen, Denmark. July 4-11, 2004]

<http://www.icme-10.dk> →

Programme → Topic Study Groups → TSG 17 → Papers

Topic Study Group (TSG) 29: The history of the teaching and the learning of mathematics, [ICME-10, Copenhagen, Denmark. July 4-11, 2004]

<http://www.icme-10.dk> →

Programme → Topic Study Groups → TSG 29 → Papers

All addresses (in drafts):

<http://web.cc.yamaguchi-u.ac.jp/~ysekigch/etc.html>

The editors welcome information about other sites.

* * *

Reports on past events



Report on the Topic Study Group 17: The role of the history of mathematics in mathematics

The aim of TSG17 was to provide a forum for participants to share their teaching ideas and classroom experience in connection with the history of mathematics, in the spirit of the 10th ICMI Study on the role of the history of mathematics in the learning and

teaching of mathematics (that led to the ICMI Study Volume *History in Mathematics Education: The ICMI Study*, J. Fauvel & J. van Maanen (eds), Kluwer 2000), and to learn about work that has been done since then.

The four sessions in this group focused on *integrating the history of mathematics in the teaching of mathematics*, in an effort to make clearer the meaning of a *historical dimension in mathematics education* and deepen the understanding of its various aspects. The *Organizing Team* was A. El Idrissi (Morocco), S. Kaijser (Sweden), L. Radford (Canada), M-K. Siu (China, co-chair) and C. Tzanakis (Greece, co-chair).

The final programme consisted of 13 presentations: 5 invited talks, 4 oral presentations and 4 paper-by-distribution contributions. Each presentation was followed by discussion among participants. In the last session an extra half-hour was devoted to a general discussion and a summary of the main points raised during the four sessions. Relevant material on the presentations has been made available on the TSG 17 webpage in the form of extended abstracts, full texts, related papers, or links to other web sites. Prospective participants were able to download material of interest to them and study it in advance. However, hard copies of material on some presentations were also available on spot. At least 64 people from 22 countries participated in this group.

All material will remain available at <http://www.icme-10.dk/> (go to *Programme*, then to *Topic Study Groups* and then follow the link to TSG17). This web site of ICME-10 will be kept online till the next ICME in 2008. In addition, work is under progress to collect full texts of these presentations as papers to be refereed. Accepted papers will appear in a forthcoming special issue of the *Mediterranean Journal of Research in Mathematics Education* published by the Cyprus Mathematical Society.

Invited talks:

C-I. Fung (China): *How history fuels teaching for mathematising: Some personal reflections*

F. Furinghetti (Italy): *History and mathematics education: A look around the world with particular reference to Italy*

M. Helfgott (USA): *Two examples from the natural sciences and their relationship to the history and pedagogy of mathematics*

J. van Maanen (The Netherlands): *History in mathematics education: FAQ and facts*

G. Waldegg (Mexico): *Problem solving, collaborative learning and history of mathematics*

Oral presentations:

G.T. Bagni (Italy): *Prime numbers are infinitely many: Four proofs from history to mathematics education*

M. Barabash & R. Guberman-Glebov (Israel), *Seminar and graduate project in the history of mathematics as a source of cultural and intercultural enrichment of the academic teacher education program*

D. Taimina (USA): *History of mathematics and mechanics in digital Reuleaux kinematic mechanism collection*

J. Tattersall, & S. L. McMurrin (USA): *Using the Educational Times in the classroom*

Papers by distribution:

R.J. Charette (USA): *Integrating the history of mathematics in the teaching of mathematics*

A.R. Garciadiego (Mexico), *Elucidating through history: The case of a well-ordered set*

C. Tzanakis (Greece): *"The ontogenetic development parallels the historical development" To what extent is this claim true, or false? Remarks and results from some case studies*

O. Yevdokimov (Ukraine): *Using material from the history of mathematics in learning by discovery*

Summary

Introducing a *historical dimension in mathematics education* involves three different areas: *mathematics*, *history* and *didactics*. Out of the presentations and discussions in this group it became clear that the following two points are most needed:

(i) There is a need to construct and develop appropriate relevant *didactical material*, which can either be used directly in the classroom or constitutes a resource for mathematics teachers. The material should aim to motivate and guide the teacher to improve the teaching approach, or understand better the students' difficulties or their idiosyncratic ways in learning mathematics.

(ii) There is a need to enrich *teachers' education* at all levels in this direction, both by introducing courses in (particular aspects of) the history of mathematics and its relation to other disciplines, and by letting them become acquainted with historically inspired material that can be, or has been used in the classroom. In this way, teachers may hopefully begin to think of a historical dimension in teaching as a possible path for improving mathematics education at all levels, and may develop confidence and trust in this endeavour.

In this perspective, the presentations in this group can be classified roughly, as follows:

(1) *Presentations focusing mainly on introducing a historical dimension in mathematics teachers' education*

(a) *Reports on specific courses in teachers' training*: *Waldegg's* presentation concerned junior high school mathematics teachers' collaborative work on problem-solving, based on historically motivated themes. *Barabash & Guberman-Glebov* reported on a sequence of activities in prospective mathematics teachers' education that aims at making history an integral part of students' education programme and allowing them to profit from this knowledge in their teaching practice.

(b) *Presentations reporting on the design of didactic material, or its implementation in practice*. *Tattersall & McMurrin* reported on the use of the (recreational) mathematical problems published in the *Educational Times*

in the Victorian era. *Taimina* reported on the rich didactic material (still under development) that comes out of a kinematic mechanisms collection developed by F. Reuleaux in the 19th century. *Yevdokimov* presented some examples from an e-learning textbook on Euclidean geometry, which elaborates on historical problems and contains related historical information.

(2) *Presentations focusing on integrating history into classroom teaching*.

Fung gave two examples enlightened by historical materials to illustrate the point of view that it is essential in mathematics education to engage students in the process of mathematising. Through two examples in optics and chemical kinetics *Helfgott* illustrated the deep interrelation between mathematics and the physical sciences and how rich and fruitful teaching ideas this can generate. *Charette* outlined historically motivated teaching capsules on elementary geometry, whereas *Garciadiago* gave a more advanced example on how history can be illuminating in the context of undergraduate teaching.

(3) *Presentations focusing on more general issues*:

van Maanen talked about questions from different quarters that ask about the role of history in mathematics education and classified them according to whom the enquirers are, what they ask and what can the feedback from such questions be. *Furinghetti* presented an outline of the different views on the role of history in mathematics education and identified two main lines of approach: (i) history as a vehicle to reflect on the nature of mathematics as a socio-cultural process; (ii) history as a possible way to conceive and understand mathematical objects. *Bagni* discussed some epistemological issues related to the historical analysis of a mathematical topic, necessary for achieving an effective and correct use of historical data in mathematics education, and presented some theoretical ideas that underline the primary importance of the correct social and cultural contextualisation. *Tzanakis* considered the quite old, but still unsettled, question of whether and to what extent the ontogenetic

development parallels the historical development in mathematics, what kind of analogies are observed and what mathematics education research can profit from investigating such analogies and reported on data from three different empirical studies.

M-K. Siu, China
C. Tzanakis, Greece

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Report on the Topic Study Group 29: The History of the Teaching and Learning of Mathematics

As a part of ICME 10, the first specialized Symposium on the history of mathematics education took place as TSG (Topic Study Group) 29 (see issues no. 55 and 56 of the *HPM Newsletter*):

The History of the Teaching and Learning of Mathematics

Organised by Gert Schubring (University of Bielefeld) and Yasuhiro Sekiguchi (Yamaguchi University), together with Hélène Gispert (France), Herbert Bhekumuza Khuzwayo (South Africa), and Hans Christian Hansen (Denmark), the main goal of the Symposium was to create a net of international communication between researchers in this field, and to establish a discussion between the various national histories and their approaches.

The TSG 29 focussed on three dimensions which proved to be dominating the present research in the various countries: transmission of reform movements and modernisations of the syllabi for mathematics in primary and secondary schools, the different aspects of the teaching practice (textbooks, teaching methods, teacher training), and the cultural, political, and social functions of mathematics instruction.

The oral contributions presented in the four working sessions (6, 7, 9, 10 July 2004) were the following:

Shinya Yamamoto, *The Process of Adapting a German Pedagogy for the Modern*

Mathematics Teaching in Japan.
Nikos Kastanis (with Iason Kastanis),
Transmissions of Mathematics into Greek Education, 1800-1840: From Individual Choices to Institutional Frames.

Kristin Bjarnadóttir, *From Isolation and Stagnation to 'Modern' Mathematics - A Reform or Confusion?*

Eileen Donoghue, *The Education of Mathematics Teachers in the United States: David Eugene Smith, An Early Twentieth Century Pioneer.*

Harm J. Smid, *Between the Market and the State: The Emergence of Mathematics Instruction and of its Teachers as a Result of State Initiative and of Pressure by the Market.*

Livia Giacardi, *From Euclid as Textbook to the Gentile Reform: Problems, Methods, and Debates in Mathematics Teaching in Italy 1867 to 1923.*

Alexander Karp, "Universal Responsiveness" or "Splendid Isolation"? *Episodes in the History of Mathematics Education in Russia.*

Taro Fujita, *The Role of Intuition in Geometry Education: Learning from the Teaching Practice in the early 20th Century.*

Gert Schubring, *The State of the Art.*

The paper by Mahdi Abdeljaouad who was unable to attend, *Issues about the status of mathematics teaching in Arab countries – elements of its history and some case studies*, was made accessible on the TSG's website.

A first considerable result of these initiatives is the first *International Bibliography on the History of Teaching and Learning Mathematics*. It is accessible by a link to the TSG on the following website:

<http://www.uni-bielefeld.de/idm/geschichte.html>

Another major outcome of the Copenhagen Symposium is that a network for research in this area was established. Its news are accessible by the same link. All those interested are invited to subscribe, and to communicate relevant papers, information, and sources.

Gert Schubring, Germany



The ASG meetings of HPM in ICME-10

During the seven days of the ICME-10 conference, the spirit of HPM has been present in various different forms. The TSG 17 chairs, Constantinos Tzanakis and Man-Keung Siu, focused on the theme of the HPM Group, as shown by the title “*The role of the history of mathematics in mathematics education*”. A session of “poster round tables” was labeled “*History of mathematics and mathematics education*”. Victor Katz, Luis Puig and Evgeny Shchepin delivered talks on the relationships between history and pedagogy and two workshops (Victor Katz with Karen Michalowicz and Avikam Gazit) dealt with the same topic. It is well known that, after the ICME Conference, the HPM Satellite Meeting took place in Uppsala, Sweden. All these events provided a good deal of information about what is going on in research and in school practice concerning the relationship between history and pedagogy. The three time-slots (four hours in total) allotted to the ASG (Affiliated Study Group) meetings were used with a slightly different aim, say, at discussing the identity of the group. The points in question were:

- To make known the origin of the group; which persons contributed to its birth and growing and which was the initial impetus for the formation of this group
- To stress once again the basic ideas underlying the activities of the group
- To outline the results of work done in the past four years and the perspectives for the future.

The *first* point was illustrated in the talk of Florence Fasanelli. The full text of her talk is published in the Proceedings of the HPM Satellite Meeting of ICME 10 in Uppsala. I think that knowing the history of education enriches the background of people working in education. This talk gave the opportunity to

all participants to remember once again John Fauvel, who has contributed to writing the first draft of the text of this talk. Since new people, including young researchers, were attending the sessions, I think that it was important for them to have such an outline of the history of the group and better understand what does working in this field mean.

The *second* point was illustrated in the talk of Ubiratan D’Ambrosio. The title itself, “*Diffusion and popularization of science and mathematics*”, explains the spirit of the session. When D’Ambrosio’s talk was set in the program of ASG meetings it was my intention to stress that the spirit of the HPM group is much more than the use of history in the teaching of mathematics; it is the *conception of mathematics as a living science, a science with a long history, a vivid present and an as yet unforeseen future*, together with the conviction that this conception of mathematics should not only be the *core of the teaching of mathematics*, but it should also be the image of mathematics spread to the outside world.

The *third* point was treated by, Jan van Maanen, co-editor of the book *History in Mathematics Education: the ICMI Study*, and Victor Katz who is currently developing an important project on the use of history in mathematics teaching.

Many colleagues have attended the ASG meetings. Some of them were old members and some were newcomers wishing to understand something about the HPM group. This group promotes the interaction of people of three different worlds - Mathematics, History, Pedagogy. Thus different souls are animating it. This complex nature may be the origin of conflicts, but, at the same time, it is the real richness and power of the group.

During the discussion, some attendants asked about the rules, which regulate the life of HPM. This is the crucial point in the management of activities of any group, like ours. Rules that are too strict are sometimes an obstacle for efficient action. On the other hand, however, the absence of precise rules may be interpreted as indications of an arbitrary and antidemocratic behavior. I

think that it was important to raise this question in the final ASG meeting and the discussion, which followed, provides the new chair with new insights for acting correctly and efficiently. As the chair during the past four years I heartily wish good luck to Constantinos Tzanakis.

References

Fasanelli, F. & Fauvel, J.: 2004, 'History of the International Study Group on the Relations between the History and Pedagogy of Mathematics: the first twenty-five years, 1976-2000', in F. Furinghetti, S. Kaijser & A. Vretblad (editors), *Proceedings of HPM 2004* (Uppsala, Sweden), 1-22.

Fulvia Furinghetti, Italy

* * *



The HPM 2004 History and Pedagogy of Mathematics Conference

The week after *ICME 10* the satellite conference of HPM was held in Uppsala. The programme committee had done a nice work and the conference was blessed with rather nice weather.

There were 120 participants and more than half of them gave talks.

For me this conference was a wonderful experience. It was the first time that I had the opportunity to participate in an HPM event and I was delighted by the friendly and encouraging atmosphere amongst the participants. We, in the organizing committee, soon found out that there is a difference of culture between mathematicians and most other scientists, in the sense that mathematicians have not yet learnt to use power-point. (This is probably for the same reason that mathematicians were also late in writing their own papers - then it was that the early word processing programs were not

good at presenting mathematical formulas - now it is power-point that can not handle mathematics well enough).

The conference lasted five and half days, and most days there were two plenary sessions in the morning and then parallel sessions in the afternoon. Wednesday was excursion day and Thursday was the day of the conference dinner.



S. Kaijser in Uppsala

The presentations covered a vast part of the history of mathematics. In time it started from the Pythagorean theory of music to such modern notions as "the minimal spanning tree" in weighted graphs, and geographically it went from the far east, including China and Japan, to the far west with mathematics and astronomy of the Mayas and the Aztecs. There were invited workshops on relations of mathematics with art, literature, music and painting and contributions on medieval mathematics from Europe and the Islamic world. There were also meetings between the very old and the very new when ancient

geometry was studied with the aid of dynamic geometric software.

There were two panel discussions, one on the use of original sources in the classroom, nicely coordinated by Evelyne Barbin, and the other on the role of proof in the teaching of mathematics - mainly concerning the interplay between proofs that explain in an Aristotelic sense and other proofs.



Exploring perspective in Uppsala

In the contributed talks the range was of course even wider. There were talks that were mainly of interest to historians, while others discussed how didactical theories apply to the conceptual framework of HPM, i.e. why we all believe that including some history of mathematics in our teaching we will teach better. A topic, which I consider to be of real importance for HPM, is the role that teaching and education has played for the development of mathematics, and some talks were of interest from that perspective.

As organizer I must however confess that the most exciting moments were the opening and the closing of the conference - in particular the closing, which was the end of half a year of preparations and three weeks of intensive activity.

Sten Kaijser, Sweden

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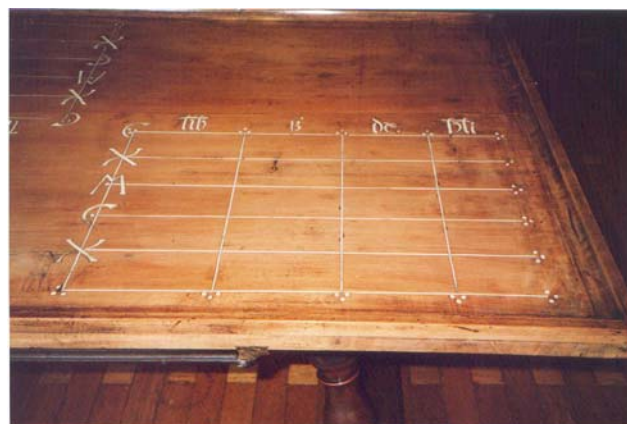
Notices

Counting tables

Have you ever seen a counting table? The question occurred to me when I came across a very fine example in Strasbourg and wondered why I had never seen one before in any museum in Europe.



Surfaces marked with lines were used for calculation with counters since antiquity and the early printed arithmetics commonly show illustrations of them being used.



The practice of using a table dedicated for the purpose continued in northern Europe long after it had died out in favour of written methods elsewhere in Europe; the table in Strasbourg appears to be as late as the end of the 16th century. My suspicion that the Strasbourg table is a rare surviving example is strengthened by the fact that it is the only one

used to illustrate counting tables in Pullan's *History of the Abacus*.



From Köbel's *Rechenbiechlin*, Augsburg, 1514

There must have been many hundreds of these tables at one time, in which case it is surprising not to find any remaining. Admittedly the Strasbourg example comes from a former Merchants' House where the wealth of the guilds is still evident. Furthermore, the lines are not cut into the surface but are made with inlaid ivory and so the table would have been thought worth preserving.

Does any reader know of other examples?

Musée de L'Œuvre Notre-Dame, Strasbourg
J. M. Pullan *History of the Abacus*, ch. IV,
Hutchinson, London: 1968

D. E. Smith *History of Mathematics* vol. II
156—192, Dover, New York: 1953

Chris Weeks, UK
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* * *

Problématique – Four Questions on the History of Proof

The Historical and Epistemological Studies are referred to as a point of view in the introductory text of the Group 4 (on

Argumentation and Proof) of the Fourth Conference of the European Society for Research on Mathematics Education [CERME 4] that will take place in Spain in February 2005, {<http://www.lettredelapreuve.it/Newsletter/04Ete/Cerme4.html>}. In this way a strong motivation is given in order to create a relevant problématique to develop some ideas and questions on the historical aspects of the proof. These ideas or questions can improve the historical understanding of the proof and its pedagogical approach.

In that context let us present a relevant problématique consisting of 4 questions.

1. What is the historical evolution of proof in algebra? And what is its evolution in school algebra?

The conceptions of proof in school algebra have been the object of a study by L. Healy and C. Hoyles {*Journal for Research in Mathematics Education*, 31(4), 2000, pp.396-428}. The historical dimension is absolutely absent in that remarkable paper. So the meaning of proof in the different epistemological contexts of the history of Algebra and of School Algebra cannot be seen. This absence can also be detected in the collective studies on the Didactic of school Algebra that have been published in recent years.

However, it is easy to see that proofs have entered impetuously in school Algebra under the influence of the New Math movement and the structural epistemology underlying it. Before that movement, proofs were absolutely absent in school Algebra, or if present they used to have a false form.

On the other hand the relevant historiography of the proof in Algebra is not empty. There are some publications on special cases, for example:

- a. Abdeljahouad, Mahdi: Proof in Arabian Algebra, *Preuve, International Newsletter on the Teaching and Learning of Mathematical Proof*, Hiver, 2002.
- b. Borowczyk, J.: Sur l'histoire des démonstrations de la règle des variations

de signe de Descartes, in *La Démonstration Mathématique dans l'Histoire*, IREM de Besançon et IREM de Lyon, 1989, pp. 275-312.

- c. Pensivy, M. : Les démonstrations de la formule du binôme au XVIIIème siècle, *ibid*, pp. 325-338.

Never the less there seems to be poverty and a lack of contextual and cognitive studies on history (or historical mentalities) of proofs in Algebra.

2. What is the place of experimentation, in particular of thought experiments in the history of proof?

The emphasis posed on the logical aspect of proof usually obscures all its other aspects. But the digital revolution produces radical changes. So we read: “The use of computers gives the mathematicians another view of mathematical reality” like “experimentation and proof in mathematics” {*Notices of the AMS*, 42(6), 1995, p. 670} or “thought-experiments: proofs in a computer environment” {*Micromath*, 10(3), 1994, p. 21}.

From this point of view a historical interest emerges for the relation of the proof with the experiments and thought experiments in particular. Relevant comments can be found in the following papers:

- a. Mueller, E.: Euclid's *Elements* and Axiomatic Method, *The British Journal for the Philosophy of Science*, 20, 1969, pp.289-309.
- b. Bettinelli, B.: Intuition et démonstration chez Archimède, in *La Démonstration Mathématique dans l'Histoire*, IREM de Besançon et IREM de Lyon, 1989, pp. 181-195.
- c. Lacatos, I.: *Proofs and Refutations. The Logic of Mathematical Discovery*, Cambridge Univ. Press, 1976.
- d. Hanna, G./Jahnke, H.N.: Another Approach to Proof: Arguments from Physics, *ZDM*, 34(1), 2002, pp. 1-8.
- e. Hattiangadi, J.: Algebra as Thought Experiment, <http://www.bu.edu/wcp/Papers/Scie/ScieHatt.htm>.

- f. Reiner, M/Leron, U.: Physical Experiments, Thought Experiments, Mathematical Proofs, Model-Based Reasoning Conference (MBR' 01), 2001, Pavia, Italy.
- g. Glass, E.: Thought- Experimentation and Mathematical Innovation, *Studies in History and Philosophy of Science, Part A*, 30(1), 1999, pp.1-19.
- h. Tall, D.: The Chasm between Thought Experiment and Formal Proof, in *Kadunz, G. et al (eds): Mathematische Bildung und neue Technologien*, Teubner, 1999, pp. 319-343.

3. Why did the notion of proof appear at all?

Until now the dominant point of view is that proof was generated in the Ancient Greek Civilization. However, not very much has been written on the reasons for this. The two most well-known justifications are those advanced by Szabó and Lloyd, respectively. According to the first, the origins of the idea of proof are to be found in the philosophy of Eleatics. He suggested that this was due to the genius of those pioneer philosophers of the 5th century {Szabó, A.: *The Beginning of Greek Mathematics*, D. Reidel, 1978, pp. 185-329}. On the other hand Lloyd regards proof to spring out of the argumentation of the rhetoric which, being an element of democracy, gives proof its social matrix (Lloyd, G.E.R.: *Magic, Reason and Experience*, Cambridge Univ. Press, 1979, pp. 59-125).

It is useful to note that without the concept of equality, together with its transitive property, it would be impossible to shape the process of proof. Equality was interwoven with the coinage system and democracy and both of them emerged and evolved within the Greek Civilization from 7th to 6th century. In other words the concept of equality was a cultural medium that played a decisive role in the evolution of the function of proof. This is a matter of fact that has escaped the attention of the aforementioned historical researches. The following paper could be useful:

Harvey, F. D.: Two Kinds of Equality, *Classica et Mediaevalia*, 26, 1965, pp. 101-146.

On the other hand, we also see an interest in the development of proof through illustrative example, as in Mesopotamia and China. Two relevant papers are the following:

- a. Chemla, K.: What is at Stake in Mathematical Proofs from Third-Century China, *Science in Context*, 10, 1997, pp. 227-251.
- b. Høyrup, J.: Mathematical justification as non-conceptualized practice: the Babylonian example, in *Proceedings of HPM 2004*, Uppsala, 2004, pp. 28-41.

4. What are the historical developments and characteristics of proof in the context of absolutist philosophies? And how did they appear in a non-absolutist context?

The historical evolution of proof is not independent of its epistemological context of different eras. Thus, looking for the cognitive changes in the thought of people who got involved in logical proofs during the historical development of mathematics, can help a deeper understanding of the notion of proof itself. Accordingly, being aware of the philosophical influences on the nature of proof can contribute to its better didactic treatment.

There are rather few such publications. We might indicate the following:

- a. Lee, Joong Kwoen: Philosophical Perspectives on Proof in Mathematics Education, *Philosophy of Mathematics Education Journal*, July 2002, <http://www.ex.ac.uk/~PErnest/pome16/docs/lee.pdf>
- b. Guichard, J.: Arrière-plans philosophiques de la démonstration, in *La Démonstration Mathématique dans l' Histoire*, IREM de Besançon et IREM de Lyon, 1989, pp. 39-52.
- c. Bkouche, R. : Quelques remarques sur la démonstration (Autour de la philosophie de Gonsseth), *ibid*, pp. 115-127.

- d. Brown, J. R.: *Philosophy of Mathematics. An Introduction to the World of Proofs and Pictures*, Routledge, 1999.
- e. Roulet, G.: The Legacy of Piaget: Some Negative Consequences for Proof and Efforts to Address Them, *Preuve, International Newsletter on the Teaching and Learning of Mathematical Proof*, May/June, 2000.
- f. Bassler, O. B.: The Surveyability of Mathematical Proof: A Historical Perspective, *Synthese*, forthcoming.

Nikos Kastanis, Greece

* * *

Monuments

On a recent visit to Barcelona, I came across some unusual mathematical monuments.

Barcelona was the south end of the meridian measurement carried out by Delambre and Méchain from Dunkirk in the 1790s to determine the length of the metre, leading to the formal definition of the metre on 10 Dec 1799. In the centre of the Barri Gotic, on the front of the Centre Excursionista de Catalunya at 10 Carrer del Paradis is a bicentennial plaque erected in 1993 to commemorate this event, stating it is 16.9 m above sea level, possibly recording a point of the original survey. Arago and Biot later extended the measurement to the island of Formentera, south of Ibiza. One of the principal streets in Barcelona is Arago, but Arago is the Catalan word for Aragon and the street is probably not named for the man.

Ramon Llull (= Ramón Llull = Ramon Lull) (1232? – 1315?), the mystic and logician, learned his Great Art in a divine illumination on Mount Randa, near Palma, Mallorca (= Majorca or Palma de Mallorca), in 1274. He martyred himself by going to Bugia (the city of Fibonacci's youth) in Algeria and proceeding to expound the errors of the local Moslem faith in the streets. He had done this twice before, barely escaping with his life each time, but this time he succeeded in getting stoned to death. He was beatified and his tomb is in the first apsidal

chapel in the church of San Francisco (= St. Francesc = Sant Francesc), in Palma de Mallorca. Though venerated as a saint, it seems unlikely he will ever be canonised, though there is a Catalan movement for this. I have seen photos of a 16C painting in St. Francesc showing Llull before the Pope at the Council of Vienna (1311-1312) – if there is any resemblance to Llull, this would be one of the earliest, probably the earliest, portraits of a mathematician.

Llull is commemorated in a stained glass window in chapel 4 (the second from the W end along the S wall) of Santa María del Mar in Barcelona. A Ramon Llull University has recently been established in Barcelona.

Near Barcelona is Montserrat, one of the most famous religious sites in the world. I was amazed to discover a large monument to Llull on a prominent point below the entrance to the site. However, the monument makes no reference to his mathematical work.

David Singmaster, England

* * *

Announcements of events

| *b s h m* |

BSHM Christmas Meeting 2004
December 11, 2004
Birmingham, UK

This meeting will take place at King Edward's School, Edgbaston Park Road, Birmingham B15 2UA
For further details, see
<http://www.dcs.warwick.ac.uk/bshm/meetings.html#xmas2004>

International Conference on History and Heritage of Mathematical Sciences,
December 16-19, 2004
Indore, India
http://www.ams.org/mathcal/info/2004_dec16-19_indore.html

The celestial geometry of John Flamsteed: mapping the heavens from 17th Century Greenwich
February 10, 2005
London, UK

A joint meeting of the BSHM and Gresham College. Admission is free.
5.30pm for 6.00pm, Barnards Inn Hall, Holborn, London EC1N 2HH

Professor Allan Chapman,
During his 46 years as Astronomer Royal, John Flamsteed elevated the mapping of the heavens to an entirely new level. His practical geometry, and his development of mathematical instruments that worked to a new standard of accuracy, meant that his Catalogue remained in use well into the 18th century. Its production, however, plunged Flamsteed into a bitter controversy with Isaac Newton, while his eminence as an astronomical geometer led to his being invited to deliver an influential lecture series at Gresham College in 1684.

Fourth Conference of the European Society for Research in Mathematics Education (CERME 4)
February 17-21, 2005
Sant Feliu de Guíxols, Spain

For details, see
<http://cerme4.crm.es/>

4th Colloquium on the Didactics of Mathematics (with international participation)
Friday 22 & Saturday 23 April 2005
Rethymnon, Crete, Greece

The Colloquium is devoted to various aspects of teaching and learning Mathematics, one of its main themes being *The didactical value of the history and epistemology of Mathematics*

Among the invited speakers are:
Fulvia Furinghetti, *The history of mathematics and teachers' education*

Gert Schubring, *Generalizing the concept of multiplication - Epistemological implications of the relation between quantity and number*

Important Dates

October 10, 2004, Submission of abstracts
December 10, 2004, Submission of full texts
January 31, 2005, Notification of acceptance

The abstracts of accepted papers will be posted on the website.
The Proceedings will be available after the Colloquium and hard copies will be sent to all participants via ordinary mail.

Contact:

Dr. C. Tzanakis, Department of Education, University of Crete, University Campus, Rethymnon 74100,
e-mail: tzanakis@edc.uoc.gr, tel: 30-28310-77629, fax: 30-28310-77596

Updated information on the programme of the colloquium and practical details can be retrieved from its website:

<http://www.edc.uoc.gr/4colloquium>

The 22nd International Congress of History of Science

24-30 July 2005

Beijing, China

The general theme is "Globalization and Diversity". Discussions will focus on the diffusion of science and technology between different cultures in the past, and its impact on the world today, as well as its prospects for the future advance of human civilisation.

The First Circular is available from the Congress Website:

<http://2005bj.ihns.ac.cn>

For further information, please contact to the Congress Secretariat.

Secretariat of the 22nd ICHS
Institute for History of Natural Science

Chinese Academy of Sciences
137 Chao Nei Street
Beijing 100010
CHINA

e-mail: 2005bj@ihns.ac.cn

6th International Symposium on the History of Mathematics and Mathematical Education using Chinese Characters (ISHME)

August 4-7, 2005

Tokyo, Japan

The 6th ISHME will call its participants to discuss general issues related to the history of mathematics and mathematical education in East Asia. In addition to these topics, the Symposium will make the mathematics in East Asia from the 16th through the 19th centuries in global network a special subject of discussion.

The First Circular is available from the Secretariat of the ISHME6.

For further information, please contact to the Congress Secretariat.

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460-1 Kamisadori
Maebashi, Gumma, 371-0816
JAPAN

e-mail: koba@maebashi-it.ac.jp

The Euler 2005 Conference

August 7-10, 2005

Portsmouth, RI, USA

The theme of the conference will be the life and work of Leonhard Euler during the 1760s.

For more information, see

<http://www.EulerSociety.org/>

Mathematical Textbooks: History, Production and Influence

September 24-25, 2005

Oxford, UK

A joint meeting of the Oxford University Department for Continuing Education and the BSHM

Rewley House Organiser: Raymond Flood
BSHM Organisers: Jackie Stedall, June Barrow-Green (j.e.barrow-green@open.ac.uk)
Further details will be available in due course.

The first Iranian workshop on the history of mathematics

October 12-15, 2005

Zirab, Iran

Held by the Iranian Mathematical Society and Shahid Beheshti University. The major items to be discussed are as follows:

- History of arithmetic
- History of geometry
- History of algebra
- History of astronomy
- History of optics
- History of mathematical entertainments and enigmas

The workshop will be held in Persian for a limited number of participants. For more information contact Dr. M.-GH. Vahidi at the Mathematics Faculty of Shahid Beheshti University (Tehran).

ICME-11

July 6-13, 2008

Monterrey, Mexico

ICME-11 will take place in Monterrey (Mexico), at the “CINTERMEX”, the convention centre of the city. There is the tradition to organize the Satellite meeting of HPM in sites ‘close’ to the venue of ICME: for example, in 1992 ICME was in Quebec city and the Satellite meeting was in Toronto (both in Canada), in 1996 ICME was in Sevilla (Spain) and the Satellite meeting in Braga (Portugal), in 2000 ICME was in Tokyo/Makuhari (Japan) and the Satellite meeting in Taipei (Taiwan), in 2004 ICME is in Copenhagen (Denmark) and the Satellite meeting in Uppsala (Sweden). We encourage the members of HPM to submit proposals for the organization of the Satellite meeting of ICME-11.

. . . this history of the embryo (ontogeny) must be completed by a second, equally valuable, and closely connected branch of thought—the history of race (phylogeny). Both of these branches of evolutionary science, are, in my opinion, in the closest causal connection; this arises from the reciprocal action of the laws of heredity and adaptation. . .



Ernest Haeckel
(1834-1919)

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Items for the Newsletter should be sent to the editors, preferably by email (see addresses below). The Newsletter appears three times a year with the following deadlines for next year.

Deadline for material	Sent to distributors
12 February 2005	1 March 2005
14 June 2005	1 July 2005
15 October 2005	1 November 2005

The Newsletter is the communication of the International Study Group on the Relations between the History and Pedagogy of Mathematics, an affiliate of the International Commission on Mathematical Instruction.

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