OPEN-ENDED PROBLEMS: A method for an educational change

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Abstract

On the problems encountered with realizing educational change will be discussed. The concept "open-ended problem" with some examples will be deal with. The rest of the talk will describe different possibilities to use open-ended problems in mathematic class.

The purpose of school education in each country is, more or less, to develop independent, self-initiative, critical thinking, motivated and many-sided skilled individuals who will manage in societal settings which they will counter later on in their life. The key question is what kind of school instruction is optimal for this goal.

A need for reform in school mathematics

Conventional school teaching has been accused that it considers the action and the context where learning happens totally different and neutral concerning the topic to be learned. However, psychological studies show that learning is strongly situation-connected (e.g. Brown & al. 1989, Collins & al. 1989, Bereiter 1990). Furthermore, recent psychological research (Bereiter & Scardamalia 1996) has confirmed the hypotheses set e.g. by Anderson (1980) that learning of facts and procedures happens through different mechanisms.

This points out that in instruction there should be offered to pupils different methods to learn, on the one hand conceptual knowledge (as facts), and on the other hand procedural knowledge (as using facts). Conventional school teaching suits very well for learning of facts, whereas learning of procedural knowledge demands pupils' self-initiated active studying. One possible solution for the latter case is offered by open learning environments, since within them one can deal with real, existing problems, be active and learn in natural settings. Since learning happens then by investigating and looking for solutions of problems, such an active studying is explained to lead to better understanding of key principles and concepts. Active learning puts pupils into a realistic and contextual problem solving environment, and thus can combine the phenomena of the real life and the class room (Blumenfeld & al. 1991).

Resistance for change

Pupils' active working in class is compatible with the constructivist understanding of learning (cf. Davis & al. 1990) which emphasizes that each individual has a characteristic way of gestalting and perceiving the world around. Additionally, the constructivism puts strongly forward that a person's knowledge will be formed and changed through his actions. Such a learning occurs at best in open learning environments, e.g. when using investigations and project works.

When trying to develop the use of open problems in mathematics teaching in school, it seems that teachers' own view of mathematics, i.e. their conceptions on good mathematics teaching, conducts very strongly their decisions on instruction. If open teaching is not in concordance with a teacher's view of teaching, the reform will not be successful, although the teacher will be trained. Therefore, the teacher's own beliefs and conceptions on teaching are in a key position.

Here we understand that an individual's *beliefs* are formed on the base of his subjective experiences and are rather stable ways of thinking which are usually emotion-laden (for more details on the definition of beliefs see e.g. Pehkonen & Törner 1996). *Conceptions* are understood as an individual's conscious beliefs for which he is usually able to give reasons. *Views* belong to conceptions, but they are more spontaneous and have more affective coloring. Beliefs do not occur totally separately, but in clusters which together form a person's belief system. When discussing on a person's mathematical belief system, we often use the concept *mathematical world view* which originates from Schoenfeld (1985).

The use of open approach requires a change in a teacher's role. The teacher is no more only a transmitter of knowledge, but a guide and a facilitator for learning as well as a planer of learning situations. In order this could be possible, usually the teacher's beliefs on teaching and learning should be changed. If his beliefs on implementation of teaching and on pupils' learning possibilities will stay conventional, he can feel pupils' active working in open learning environments problematic, since then several different actions happen simultaneously in the classroom. Especially a less experienced teacher might feel the unorder as a threath which he is not able to deal with. (Blumenfeld & al. 1991)

A new opportunity: "use of open-ended problems"

When looking for a new teaching method which might confront the challenges set by constructivism, the so-called open approach has been developed in the 1970's in Japan (e.g. Nohda 1991, Becker & Shimada 1997). Internationally it is accepted that open-ended problems form a useful tool when developing mathematics teaching in school in such a way which emphasizes understanding and creativity (e.g. Nohda 1991, Silver 1993, Stacey 1995). Papers from a larger group of international specialists are collected and published in a report (Pehkonen 1997).

Tasks are said to be *open*, if their starting or goal situation is not exactly given (cf. Pehkonen 1995b). Thus pupils are left freedom in solving the task which in practice means that they may end with different, but equally right solutions, depending on their additional selections and emphasis done during their solution processes. Therefore, open tasks have usually several right answers. When using open tasks in mathematics teaching, pupils have an opportunity to act like a creative mathematician (cf. Brown 1997). Open-ended problems are such open tasks which can be counted as problems.

What are Open-ended Problems?

Several types of problems are collected under the title ”open problems” (cf. Pehkonen 1995b): *investigations* (where a starting point is given), *problem posing* (or problem finding or problem formulating), *real-life situations* (which have their roots in the everyday life), *projects* (are larger study entities, requiring independent working), *problem fields* (or problem sequences or problem domains; a collection of contextually connected problems), *problems without a question*, and *problem variations* (”what-if”-method). Several examples of different types of open problems can be found e.g. in the published papers of Nohda (1991), Stacey (1995), Silver (1995) and in the edited collection of Pehkonen (1997). Some mathematics educators use the word ”exploratory” as a synonym for ”open” (e.g. Avital 1992), in order to prevent confusion with the unsolved problems of mathematics (cf. also Silver 1995).

As an example of open-ended problems, we will consider two examples of problem fields: Polygons with matchsticks, and Number Triangle. They represent those problems I have developed to be used in heterogeneous classes in Finnish comprehensive schools. In each problem field, the difficulty of the problems ranges from very simple ones that can be solved by the whole class, to harder problems which only the more advanced students might be able to solve.

One of the characteristics of problem fields is that they are not bound to a fixed class-stage, but are suitable for mathematics teaching from primary level to teacher in-service education. The role of the easier problems in problem fields is especially to reinforce the problem solving persistence of pupils. The most important aspect of all in these problems is the way in which they are introduced to a class: <u>The problem field ought to be given gradually to pupils, and the continuation should be related to the pupils’ solutions</u>. Instead of the answers and results which are not given here, the process of problem solving is of paramount importance. The most important aspect is the use of pupils' own creative power. The level to which the teacher takes the problem field, depends on the pupils’ answers.

Polygons with matchsticks

Twelve matchsticks (or cocktail-sticks, etc.) will be needed to concre-tisize the problems. The starting situation is the following:

With twelve matchsticks one can make a square (Fig. 1) the area of which is 9 au (au = area units).

From this situation has been developed a sequence of problems (a problem field). Firstly, we will choose another area, but have the perimeter of the polygon constant. Thus, in each problem the perimeter of the polygon should be made up of 12 matches

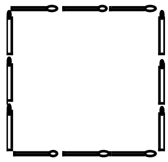

Fig. 1: A square with 12 matches.

- *Can you use twelve matches to make a polygon with an area 5 au?*

If we are willing to give more thinking time to slower pupils, the faster ones can be asked to find another (perhaps also a third) solution. Fig. 2 shows some of the pupils’ solutions.

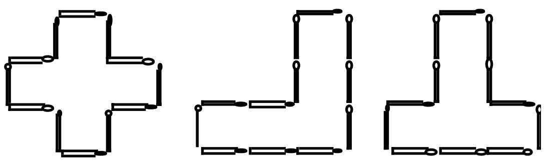

Fig: 2. Some polygons with the area 5 au.

As the next question, we might ponder the number of different solutions.

- *How many different polygons of 5 au can you make with twelve matches? Can there be more than ten different solutions?*

The pupils will probably find many of the solutions. But there are still more complicated solutions which they probably will not find. The following stage might be the comparison of the different solutions found by pupils. How many of all different solutions can be found when the whole class is working together?

Another easier direction to vary the problem is to change again the area.

- *Is it possible to use twelve matches to make a 6 au (or 7, 8 au) polygon?*

The solutions in Fig. 3 can be found easily. But are there any other solutions in each case? And how many different ones?

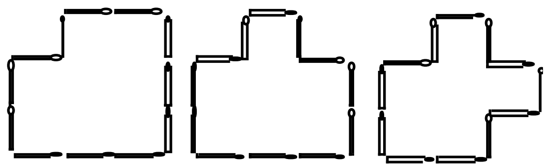

Fig. 3: Polygons with the area 8, 7, 6 au.

The method of cutting out a corner from a rectangular polygon, as in the Fig. 2 and 3, is successful until the area 5. But the question of smaller areas is more complicated, since we are compelled to change the method.

- *Is it possible to use twelve matches to make a 1 au (or 2, 3, 4 au) polygon?*

With the aid of the Pythagorian theorem, one can construct polygons with an area of 4 au and 3 au. It should also be possible to find the general solution: the parallelogram. But the question of different solutions and their number in the case of area 2 au (or 1 au) is according to my experience really hard one.

Still one enlargement of the problem field is to ask for greater polygons than area 9 au:

- *Using twelve matches is it possible to make a polygon whose area is greater than 9 au?*

This seems to be a hard one, since also in teacher pre-service and in-service courses, this question has so far not been solved.

Teaching experiences. Within the last ten years, I has worked through the problem field “Polygons with matchsticks” with many groups of teachers both on pre-service and in-service training courses as well as with some school classes in the lower secondary school.

Usually, the problem field has taken about 30 minutes. In the teacher groups, we have found many polygons with areas of 4, 5, 6, 7, 8 au. But those with areas smaller than 4 au seem to be very complicated to construct. Only in a couple of groups has somebody produced the general solution: a parallelogram. When all new ideas for solutions from the group have tried up, I have set the last problem worked on by the group as homework. My main reason for introducing problem fields in teacher training was to describe to teachers how to deal with a problem field and to give them an idea how pupils feel when solving them.

In the autumn 1996, I had an opportunity in Jena (Germany) to work with a group of highly talented pupils from the local upper secondary school. Among others, we worked through this problem field, and the pupils found the problems with smaller areas (smaller than 5 au) very challenging. Also for them, it took some time to find out other solutions than the parallelogram.

Number Triangle

The starting situation for this problem field is given in Fig. 4:

A triangle where the corner are free and some numbers are fixed on the sides.

The first problem is, as follows:

- *What numbers should be placed in the blank circles in the triangle in Fig. 4 so that the sum of the three numbers on each side is equal?*

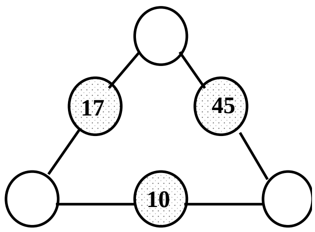

Fig: 4. Number Triangle problem.

Since there are many solutions for this problem, we will usually ask as next:

- *Can you find another solution?*

and

- *How many different solutions could there be?*

There are infinite number of solutions, but pupils usually do have difficulties in finding more than one.

Another direction might be to enlargen the number domain accessable. As a rule, pupils suppose that the numbers placed into empty circles are natural numbers.

- *Is it possible to use negative numbers in the circles?*

At the first glance, pupils in the lower secondary school may answer this question in the negative, but they usually find after working through some examples (e.g. as a home work) that it is possible.

By fixing the side sum, one gets a different kind of problem. If you want your pupils to practice with negative numbers, you may put zero (or some negative number) for the sum.

- *Can you find a solution where the triangle's side sum (i.e. the sum of the numbers on the same side) will be 80?*

In the lower secondary school, pupils usually solve this kind of problem by trial-and-error method. If they don't find out any general rule, this might be a good place to practice systematic trial-and-error.

An interesting enlargenment is to ask for the possible number domain:

- *Which numbers are possible as the triangle's side sum?*

Most pupils will suggest here integers. After negotiations, they might see the possibilities of fractions, but irrational numbers seem to be unpossible for them to think and invent.

The usual question of generalisation may be discussed also within this problem field:

- *How could you generalise the problem?*

School experiences. In the fall of 1987, I experimented with the use of Number Triangle as a separate problem in one grade 7 class in Helsinki. My original objective was to improve the pupils' mental calculation skills in an unusual way, as well as their problem solving skills. But the Number Triangle led to an interesting problem sequence (a problem field) which is described above, in the frame of which pupils were training among other things calculations with negative numbers for some weeks. The whole problem field was dealt with in six different lessons, but usually only as a part of the lesson. The entire class was eager to find solutions for separate problems and to search for general solving principles. Some pupils were telling me their new solutions when they met me outside of mathematics class. If we did not do the problem field during some lesson, they asked when we would continue and were willing to show the solutions they had found.

Research-based knowledge on open-ended problems

The research projects realized by the author during last ten years are so closely connected with each others that one can speak about a research program. The ultimate goal of the program is to develop mathematics teaching in the lower secondary school in Finland through open tasks. These are used in the form of problem fields; some examples one can find i.a. in Pehkonen 1995b, 1997.

The project "Open Tasks in Mathematics"

The three-year project "Open Tasks in Mathematics" which is described briefly in Pehkonen (1995a), was carried out during the years 1989-92 in Helsinki (Finland) and was supported financially by the Academy of Finland. The project was implemented in grades 7-9 (i.e. 13-15 year-old pupils) in lower secondary school, and concentrated on the use of problem fields (a certain type of open problems) in addition to the conventional mathematics teaching.

In the pilot study phase during 1987-89, the research design was tested, indicators were developed and the problem material was elaborated into its final form. The main experiment began in the fall of 1989 in Helsinki with ten grade 7 classes, and continued with those classes through the whole lower secondary school (up to grade 9), i.e. to spring 1992. Half of the classes formed an experimental group, and the other half a control group.

The project description. The purpose of the research project was to develop and foster methods for teaching problem solving (in the sense of open problems) in lower secondary schools. We tried to stay within the frame of the "normal" teaching, i.e. in the frame of the valid curriculum, and to take into account the teaching style of the co-operating teachers when using problem fields. The objectives of the research project can be categorized into five fields of emphasis:

(1) To clarify possibilities and methods for the use of problem fields in teaching.
(2) To foster pupils' attitudes against mathematics and mathematics teaching.
(3) To develop the flexibility of pupils' thinking.
(4) To examine how pupils' problem solving ability develops when normal teaching methods are used.
(5) To develop teachers' conceptions of mathematics and mathematics teaching.

Both in the beginning and at the end of the experimental phase, teachers' and pupils' conceptions of mathematics teaching have been gathered using questionnaires and interviews. In the main experiment, the experimental group and the control group differed in the point that from the mathematics lessons of the experimental group about 20 % (i.e. once a month about 2-3 lessons) was reserved for dealing with problem fields. There was a questionnaire for each problem field in which the pupils' conceptions of using that problem field were ascertained. The teachers' conceptions of using problem fields were obtained with short interviews after each term. The teachers in the control group were told that they are participating in an experiment, whose aim was to investigate the development of pupils' problem solving skills in natural teaching environment. They were not told anything about problem fields nor the experiment group. Pupils in both groups solved in their class work some open problems which were the same for both.

Results. Summarizing the results in respect of pupils in the research project (for more details see e.g. Pehkonen 1995a), one may state that the pupils experienced the problem fields used as an interesting form of learning mathematics. They liked very much most of them, and were motivated and activated also during other parts of mathematics lessons. Their mathematical views did not change statistically significantly during the three years of the experiment. But the non-significant changes in the questionnaire data, in the classroom observations, and in their teacher's evaluations indicate that there existed a change, and the change was in most cases positive.

Based on the research findings concerning teachers (for more details see e.g. Pehkonen 1993), some questions arose: On which reasons do the teacher actually form his assessment of the selection of an open-ended problem (or more generally of mathematical teaching material)? It seems that some of the teachers based their assessment on the convenience to use the material. Which kind of objectives should we pose for those conducting the change in teaching? In the research project, we aimed with open problems (problem fields) to cause change in mathematics teaching. In the research findings, we noticed that about one third of the reasons given by the teachers were connected with the convenience to use the material. Thus in order to reach change with the aid of teaching material, one may choose between, at least, two strategies: (1) One emphasizes the pupil-centerness and the mathematical content of the tasks. This leads to the problem of teacher in-service training. (2) One is satisfied with the offering of easy-to-use materials to teachers. This leads to the problem of producing material.

Summarizing, the results suggest that the open approach, when used in addition to the conventional teaching methods, seems to be a promising one. The pupils preferred this kind of mathematics teaching where one important factor was the freedom let to pupils to decide their own learning rate. The use of open problems, in the form of problem fields, has been experienced to be a so promising approach that there is now a published mathematics textbook for grades 7-9 using this approach (cf. Espo & Rossi 1996).

The project "Teachers' Conceptions on Open Tasks"

Another hindering aspect of the implementation of the problem solving approach - teachers' pedagogical content knowledge - was taken here into the focus. The research project (#39375) "Teachers' Conceptions on Open Tasks" was financially supported by the Academy of Finland, and implemented during 1998. Its purpose was to clarify what are teachers' possibilities to apply the principles of open teaching in his daily instruction, especially in the form of open tasks, i.e. to find out teachers' pedagogical content knowledge in the case of open tasks.

The data was gathered with several methods: A questionnaire (statistical data) as well as interviews and observations (qualitative data) were used. But the main method for data gathering was a postal survey. The information obtained with the questionnaire was checked and completed with interview and observation data. The subjects of the postal survey were teachers in the lower secondary school in Finland. From all the Finnish lower secondary schools, it was selected at random every sixth (N = 135). A letter with a questionnaire was sent to the directors of those schools, asking them to give the questionnaire to the teacher of the class 8A. The number of returned filled-in questionnaires was about one half of the original sample (N = 74). The data was analyzed according to the phenomenological principles.

As an answer to the first research question of the project: "What kind of conceptions do teachers possess on open tasks and on their role in instruction?", we can state the following (Pehkonen 1999): The results obtained showed that about a half of the teachers responding were not able to formulate a proper definition for open tasks. On one hand we have good reasons to believe that most of those teachers who have not responded belong to this group, and therefore, approximately only one quarter of the Finnish lower secondary school teachers know the term "open task". More on the preliminary results can be found in Pehkonen (1999).

Discussion

The results of the first project "Open Tasks in Mathematics" showed that open tasks do have a clear role in motivating pupils (Pehkonen 1995a). Furthermore, the results of the project showed that such an approach will function in classroom, although resistance for change could be observed in the case of some teachers and pupils (Pehkonen 1993, 1995a).

Research on teachers' and pupils' mathematics-related beliefs has revealed some mechanisms in such a change resistance (e.g. Pehkonen 1994, Pehkonen & Törner 1996). Through the results of the project "Development of Pupils' Mathematical Beliefs", one was better able to understand the inertia forces of the change: i.a. pupils' beliefs (e.g. Hoskonen 1998) and their changes (e.g. Hannula 1998a, 1998b), as well as teachers' beliefs (e.g. Pehkonen 1998, 1999). These all research projects have formed a basis to develop and to confirm the model of Active Mathematics Learning (the AML model) for the mathematics instruction of the comprehensive school.

The aim of the AML model is that every pupil will reach strong calculation skills and sure mathematical understanding. Therefore, mathematics teaching on every grade level should contain the following components in a suitable proportions (perhaps, in 2:1):

(1) Calculation skills: Every pupil should learn the basic calculation skills to the level of automacy.

(2) Problem solving skills: Every pupil should gather his own experiences in problem solving, e.g. by solving open problems.

Concluding comments

There are still some important questions which need further research. The results of the research project ”Open Tasks in Mathematics” revealed that teacher change is not an easy problem to solve. Although the cooperating teachers were voluntary, the adaptation of a new teaching strategy was not unproblematic. Therefore, key questions which should be investigated are, as follows: What are the obstacles of change for teachers? How to remove these obstacles of teacher change, when one has been able to gestalt them? Could it be true that all teachers are not able to change because of their personality characteristics?

The model of Active Mathematics Learning (AML) should further be sharpened with the methods of research. The next step might be to clarify what all can be learned using the AML model. A central research topic might be the comparison of learning results between the AML model and the conventional teaching model. In the focus of comparison, there might be i.a. the following points: 1) cognitive school achievements, 2) affective school achievements, 3) higher level thinking skills (i.a. understanding, problem solving skills), 4) data gathering and processing skills, 5) transfer of learning. As a hypotheses, one could present a belief that the use of the AML model would improve learning results especially in points 2) - 5) and that in point 1), the results are not, at least, worse than when using the other model. Furthermore, it would be of interest to clarify, whether there are any gender differences in points 1) - 5).

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