History of Mathematics and Education

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History and Epistemology for Mathematics Education: An Ever-Increasing Interest

In the last 30 years or so, integrating history of mathematics (HM) in mathematics education (ME) has emerged as a worldwide intensively studied area of new pedagogical practices and specific research activities. However, the interest on the HM in the context of ME dates back to the second half of the nineteenth century. Mathematicians like F. Klein and A. De Morgan and historians like P. Tannery and G. Loria showed an active interest on the role of the HM in education. Already at that time, history enters into textbooks, e.g., in France in Rouché and Comberousse (Barbin et al. 2008, ch. 2.2.). At the beginning of the twentieth century, this interest was revived as a consequence of the discourse and the related debates on the foundations of mathematics. Poincaré criticized Hilbert's axiomatic approach and declared that the history of science should be the "principal guide for the educator." Later, history became a resource for the various epistemological approaches, like Bachelard's *historical* epistemology (Bachelard 1938), Piaget's genetic epistemology (Piaget and Garcia 1989), and Freudenthal's *phenomenological* epistemology (Freudenthal 1983), at the same time stimulating the formulation of specific ideas and conclusions on the learning process (Lakatos 1976; Brousseau 1997; Ernest 1994) (see ▸ Learning Study in Mathematics Education).

The interest on the history and epistemology of mathematics became stronger and more competitive in the 1960s and 1970s in response to the "New Math" reform. Those supporting the reform were strongly against "a historical conception of education" ("à bas Euclide" declares Dieudonné), whereas, for its critics, history appeared like a "therapy against dogmatism," conceiving mathematics not only as a language but also as a human activity.

Since 1968 ME has constituted a standard subject in regularly organized international meetings. In 1972 a working group on the "History and Pedagogy of Mathematics" was organized by Ph.S. Jones during the 2nd *International Congress on Mathematics Education* (ICME 2), and in 1976 the *International Study Group on the relations between the History and Pedagogy of Mathematics* (known afterwards as the *HPM Group*) was created as an international study group affiliated to the *International Commission on Mathematical Instruction* (ICMI). For the history and the activities of this group, which has been playing a leading role in this area, see Fasanelli and Fauvel (2006).

In fact, the eight points which constituted the original focus and aim of the HPM Group and to some extent achieved so far, are still pertinent today (see Fasanelli and Fauvel 2006) to promote international contacts and exchange information in this area, to promote and stimulate interdisciplinary investigation, to further a deeper understanding of mathematics' evolution, to assist in improving instruction and curricula by relating mathematics teaching and its history to the development of mathematics, to produce relevant material for the teachers' benefit, to facilitate access to this material and to historical sources, and to promote awareness of the relevance of the HM for mathematics teaching and its significance for the development of cultures.

In the mid 1980s, the French network of the IREMs (*Instituts de Recherche sur l'Enseignement des Mathématiques*) began to organize every 2 years a Summer University on the History and Epistemology in Mathematics Education. Since 1993, this was extended on a European scale constituting the *European Summer University on the History and Epistemology in Mathematics Education* (ESU), which gradually has become a major international activity in the spirit of the HPM Group (Barbin et al. 2010). This spirit goes beyond the use of history in teaching mathematics and conceives mathematics as a living science with a long history, a vivid present and an unforeseen future, together with the conviction that this conception of mathematics should be not only the core of its teaching but also its image spread out to the outside world.

The gradually increasing interest of mathematicians, historians and mathematics teachers and

educators in this area, has led to various research activities and didactical experiments, which were analyzed, and their results were disseminated in the context of regularly organized local and international meetings and were presented in numerous publications in international journals, collective volumes, and conference proceedings. Some standard works in chronological order (with detailed extensive bibliography therein) are NCTM 1969/1989, Commission Inter-IREM (1997), Swetz et al. (1995), Calinger (1996), Fauvel and van Maanen (2000), Katz (2000), Bekken and Mosvold (2003), Katz and Michalowicz (2004), Barbin and Benard (2007), Knoebel et al. (2007), Barbin et al. (2010), and Katz and Tzanakis (2011).

There are three different – though interrelated – types of contributions of didactical/educational research and the associated experimental work on the role of HM in ME in the last 30 years, epistemological, cultural, and didactical.

Epistemological Contributions

Bachelard and Lakatos' influence clearly appears in the research conducted on the role of problems and on constructing and rectifying concepts and theories. This research should be placed both in the context of pedagogical constructivism of the 1980s and 1990s, as well as, in the area of "problem solving" (see ▸ Problem Solving in Mathematics Education). Here, history plays a crucial role, since it provides specific pertinent examples of problems on the basis of which concepts were invented and/or transformed (see, e.g., Commission Inter-IREM 1997; Fauvel and van Maanen 2000, section 7.4.7; Katz and Michalowicz 2004).

More generally, history allows for a deeper analysis of mathematical activities, thus motivating and stimulating research in relation to "activity-based teaching," promoted in the 1990s. A lot of research work in this context consists of determining the issues at stake and the practices adopted concerning mathematical reasoning. They show that rigor and the evaluation of mathematical proof have been subjected to debate and controversy among mathematicians. Actually, fundamental notions like rigor, evidence, and proof have been different in different historical periods; that is, (meta) ideas and (meta) concepts that today are taken for granted in their present form are actually the product of a historical development; there is a *historicity* inherent to them and maybe it is more appropriate to use plural number when referring to them (Barbin and Benard 2007). This fact gave rise to ideas about learning processes of school mathematics. From this point of view, history has what has been called a "*replacement role*" ("*rôle vicariant*" in French) by offering to teachers the possibility to approach and explore pieces of mathematics, which are not included in the official school curricula, and in this way to often replace what is usual with something different and/or unusual.

Since the 1990s, a lot of research has been conducted on number systems, equations, geometrical constructions, the role of technical instruments in mathematics, the history of proof, etc. (Calinger 1996). In addition, the historical study of the range of applicability of concepts has led to a critical analysis of school programs (e.g., on the history of probability and statistics; see Barbin 2010; Katz and Tzanakis 2011, ch. 16).

More recently, many works propose to connect history and semiotics in order to analyze the role of script and figures in the evolution of mathematics, concerning both invention of concepts and the mode of reasoning (see ▸ Mathematical Proof, Argumentation, and Reasoning). There are several international meetings and publication in this context (see, e.g., Hanna et al. 2010).

Cultural Contributions

In many works it is claimed that the main cultural aspect of history is to provide a different image of mathematics both to teachers and – more importantly – to students, on which their more positive relation with mathematical knowledge can be founded. In fact, history allows placing mathematics in the philosophical, artistic, literary, and social context of a certain period.

Thus, teachers could link mathematics to philosophy, or history; e.g., the history of the concept of perspective, which is also interesting for the teachers of plastic arts, stimulated many works (e.g., Commission inter-IREM 1997). Similarly, the relation between mathematics and literature leads to cultural insights if seen in a historical perspective. This could consist of the intrusion of mathematics into a roman through this roman's human characters, but mathematics could also inspire the subject or the structure of a roman.

Other research activities have shown the way HM leads to the history of science. In particular, reading a text often requires placing it in relation to the author's scientific preoccupations, prejudices, and concerns. Sometimes, the solution of a problem requires establishing passages or developing analogies among different disciplines. It is interesting for ME to study the circulation of problems, concepts, methods, or modes of writing (scripts) between mathematics and other sciences (e.g., see the work on vectors Barbin 2010).

Research on the history of complex numbers constitutes a privileged domain to unfold the different aspects of cultural or interdisciplinary development (see ▸ Interdisciplinary approaches in Mathematics Education). This case articulates and connects mathematics to physics and philosophy; additionally, it questions mathematical invention and the link between reality and the status of mathematical truth (Fauvel 1990; Barbin 2010).

In the last few years, the relation between mathematics and other disciplines has been subsumed in education by the concept of modelization. On this issue, proposals have been put forward that seem to be incompatible with each other if one ignores the different conventions adopted for this concept in its short history. The conception of mathematics as an "experimental science" – also used in education – has given rise to historical reflections, e.g., on the comparison between mathematical and physical experiments.

This multifaceted aspect of the character of mathematical notions and concepts revealed through the work done on the HM supports the idea for an interdisciplinary teaching that has been promoted since 2000. The movement for interdisciplinarity has been officially incorporated in curricula through pedagogical innovations in secondary education, and in this context the interest in the history of science has been stimulated (Barbin et al. 2010).

There are two areas which have been recently developed on the intersection among mathematics, culture, and societies: on the one hand, the history of ME, which forms part of the HM in general, has contributed to research in education proper and has led to several international meetings (see ▸ History of Mathematics Teaching and Learning); on the other hand, the research on ethnomathematics initiated by U. d'Ambrosio makes appeal on history, given that the investigated methods and practices can be traced back to old ones that were transmitted to the present era (see ▸ Ethnomathematics).

History of Mathematics in the Classroom

There is a gradually increasing number of works introducing a historical aspect in the mathematics classroom (Fauvel 1990); as a consequence, the activities and publications in the context of the HPM Group have been enriched. A comprehensive presentation is given by Fauvel and van Maanen (2000); for subsequent developments, see Katz (2000), Katz and Michalowicz (2004), Shell-Gellasch and Jardine (2005), Knoebel et al. (2007), and Katz and Tzanakis (2011). This does not mean that the research conducted concerns a line of approach of teaching history to students as an independent subject, but rather, to orient the teacher towards enriching his/her teaching by taking into account ideas based on epistemology and history or directly introducing historical elements. The aim of "introducing a historical perspective in mathematics teaching" is not to approach a subject in the classroom or at home in a way completely detached from conventional teaching. Rather, it should be meant as the stimulation of historical or epistemological reflections of the teacher in connection with his/her teaching (Barbin 2010)

to give important dates for a concept, to explain its historical significance, to refer and/or read original texts, to solve "historical problems," etc.

Reading original texts allows for a "cultural shock" by directly immersing mathematics into history. Therefore, the majority of researches insist on the necessity to read original texts, not in relation to our present knowledge and understanding, but in the context where they were written. It is this line of approach which becomes a source of "epistemological astonishment" by questioning knowledge and procedures that "have been taken for granted" so far. Thus, reading original texts has a strong virtue of "*reorientation*" ("*vertu dépaysante*" in French). A lot of works in the last 20 years present numerous resources for reading original texts and the variety of activities related to this reading. It gives the opportunity to introduce methods that may not be taught today and/or to compare different methods of solution (e.g., Fauvel and van Maanen 2000; Knoebel et al. 2007; several chapters especially in Swetz et al. 1995; Barbin et al. 2010; Katz and Tzanakis 2011).

The introduction of a historical dimension in ME requires appropriate teacher training, however (Fauvel and van Maanen 2000, ch. 4). Since the creation of the HPM Group, a large number of studies have been devoted on the conditions of such preservice and in-service training. To this end, a large number of monographs and anthologies addressed to students and teachers have appeared in the last 30 years. A direct approach in this context – though not the only one – is to give undergraduate courses based on historical material (e.g., Katz and Tzanakis 2011; Knoebel et al. 2007).

The analysis of numerous teaching experiments has led to specific pedagogical and didactical questions. On one hand, such experiments – like any other pedagogical innovation – aim to have a value by themselves. But, they are not easily reproducible since they depend on the teacher's culture and the resources at his/her disposal. On the other hand, these experiments should be evaluated/assessed in relation to their own objectives, which do not often correspond to the conventional conceptions of evaluation and assessment. These two issues constitute the starting point of new ideas and trends on developing further a historically inspired and epistemologically driven approach to teach specific pieces of mathematics and/or to design mathematics curricula.

Cross-References

- Ethnomathematics
- History of Mathematics Teaching and Learning
- Interdisciplinary Approaches in Mathematics Education
- Learning Study in Mathematics Education
- Mathematical Proof, Argumentation, and Reasoning
- Problem Solving in Mathematics Education

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